(10) Calculeu els límits següents, si existeixen:

(a) $$\lim_{x\to 0} \frac{2x-\sin x}{\sqrt{1-\cos x}} = \lim_{x\to 0} \frac{(2x-\sin x)/x}{\sqrt{1-\cos x}\,/x} = \lim_{x\to 0} \frac{2-\frac{\sin x}{x}}{\sqrt{\frac{1-\cos x}{x^2}}\cdot\frac{|x|}{x}} =$$

$\sqrt{x^2}=|x|$

$$= \lim_{x\to 0} \frac{(2-1)}{\sqrt{1/2}} \frac{x}{|x|} = \sqrt{2}\lim_{x\to 0}\frac{x}{|x|} = \nexists$$

(b) $$\lim_{x\to 0} \frac{1}{3+2^{1/x}} = \left\{\begin{array}{l} \lim_{x\to 0^+} \frac{1}{3+2^{1/x}} = \frac{1}{3+2^{+\infty}} = 0 \\ \lim_{x\to 0^-} \frac{1}{3+2^{1/x}} = \frac{1}{3+2^{-\infty}} = \frac{1}{3} \end{array}\right\} = \nexists$$

(c) $$\lim_{x\to 0} \frac{1+2^{1/x}}{3+2^{1/x}} = \left\{\begin{array}{l} \lim_{x\to 0^+} \frac{1+2^{1/x}}{3+2^{1/x}} = \lim_{x\to 0^+} \frac{2^{-1/x}+1}{3\cdot 2^{-1/x}+1} = 1 \\ \lim_{x\to 0^-} \frac{1+2^{1/x}}{3+2^{1/x}} = \frac{1}{3} \end{array}\right\} = \nexists$$

(d) $$\lim_{x\to 2} \frac{\sqrt{7+x}-3}{x^2-4} = \frac{0}{0} = \lim_{x\to 2} \frac{(7+x)-9}{(x^2-4)(\sqrt{7+x}+3)} = \lim_{x\to 2} \frac{1}{(x+2)(\sqrt{7+x}+3)} = \frac{1}{24}$$

(e) $$\lim_{x\to 0} \frac{|x+1|-|x-1|}{x} = \lim_{x\to 0} \frac{(x+1)-(1-x)}{x} = \lim_{x\to 0} \frac{2x}{x} = \lim_{x\to 0} 2 = 2.$$

Només ens cal controlar els valors de $|x+1|$ i $|x-1|$ quan $x \simeq 0$.

(f) $$\lim_{x\to 3} \frac{\sqrt{x^2-2x-3}}{\sqrt[3]{x^2-3x}} = \frac{0}{0}$$

$$\begin{array}{c|ccc} & 1 & -2 & -3 \\ 3 & & 3 & 3 \\ \hline & 1 & 1 & \boxed{0} \end{array}$$

Ruffini: $x^2-2x-3=(x-3)(x+1)$

Per tant, observem que com $x^2-2x-3<0$ si $x\in(-1,3)$, llavors el límit només té sentit per la dreta, ja que sinó $\sqrt{x^2-2x-3}$ no està ben definida. Així:

$$\lim_{x\to 3^+} \frac{\sqrt{x^2-2x-3}}{\sqrt[3]{x^2-3x}} = \lim_{x\to 3^+} \frac{(x-3)^{1/2}(x+1)^{1/2}}{(x-3)^{1/3}\cdot x^{1/3}} = \lim_{x\to 3^+} (x-3)^{1/6}\frac{(x+1)^{1/2}}{x^{1/3}} = 0.$$