(4) Considerem les funcions $f(x) = \frac{x+1}{x-1}$, $g(x) = \frac{x-3}{x+5}$.

Determinem les funcions inverses, $f^{-1}(x)$, $g^{-1}(x)$, les composicions $(g \circ f)(x)$, $(g^{-1} \circ f)(x)$ i als dominis corresponents.

- $f(x) = y \Leftrightarrow \frac{x+1}{x-1} = y \Leftrightarrow x+1 = y(x-1) \Leftrightarrow x(1-y) = -1-y \Leftrightarrow x = \frac{1+y}{y-1}$

  Així $f^{-1}(y) = \frac{1+y}{y-1}$.

- $g(x) = y \Leftrightarrow \frac{x-3}{x+5} = y \Leftrightarrow x-3 = y(x+5) \Leftrightarrow x(1-y) = 3+5y \Leftrightarrow x = \frac{3+5y}{1-y}$

  Així $g^{-1}(y) = \frac{3+5y}{1-y}$

- $(g \circ f)(x) = g(f(x)) = g\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1} - 3}{\frac{x+1}{x-1} + 5} = \frac{x+1-3(x-1)}{x+1+5(x-1)} = \frac{-2x+4}{6x-4} = \frac{-x+2}{3x-2}$

- $(g^{-1} \circ f)(x) = g^{-1}(f(x)) = g^{-1}\left(\frac{x+1}{x-1}\right) = \frac{3 + 5\frac{x+1}{x-1}}{1 - \frac{x+1}{x-1}} = \frac{3(x-1)+5(x+1)}{x-1-(x+1)} = \frac{8x+2}{-2} = -4x-1$

- $D_f = \mathbb{R} \setminus \{1\}$, $D_g = \mathbb{R} \setminus \{-5\}$, $D_{f^{-1}} = \mathbb{R} \setminus \{1\}$, $D_{g^{-1}} = \mathbb{R} \setminus \{1\}$.

- $D_{g \circ f} = \{x \in \mathbb{R} : \underbrace{x \in D_f}_{x \neq 1} \text{ i } \underbrace{f(x) \in D_g}_{\frac{x+1}{x-1} \neq -5 \Leftrightarrow x \neq 2/3}\} = \mathbb{R} \setminus \{1, 2/3\}$.

- $D_{g^{-1} \circ f} = \{x \in \mathbb{R} : \underbrace{x \in D_f}_{x \neq 1} \text{ i } \underbrace{f(x) \in D_{g^{-1}}}_{\frac{x+1}{x-1} \neq 1 \leftarrow \text{ mai } \frac{x+1}{x-1} \text{ és } = 1 \text{ !!!}}\} = \mathbb{R} \setminus \{1\}$.