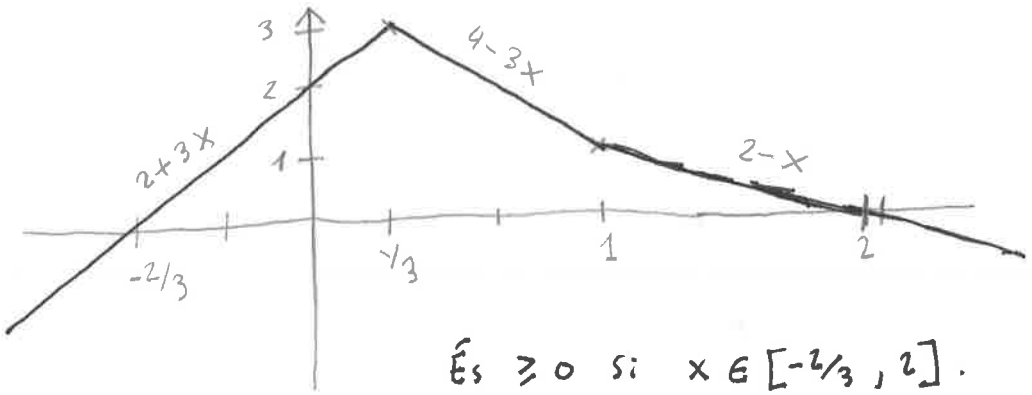
(2) Valors absoluts. Determinem els $x \in \mathbb{R}$ definits per les expressions següents:

(a) $3 - |2x - |x-1|| \geq 0$

$$3 - |2x - |x-1|| = \begin{cases} 3 - |2x - (x-1)|, & \text{si } x \geq 1 \\ 3 - |2x - (1-x)|, & \text{si } x \leq 1 \end{cases} = \begin{cases} 3 - |x+1|, & \text{si } x \geq 1 \\ 3 - |3x-1|, & \text{si } x \leq 1 \end{cases} =$$

$$= \begin{cases} 3 - (x+1), & \text{si } x \geq 1 \\ 3 - (3x-1), & \text{si } \frac{1}{3} \leq x \leq 1 \\ 3 - (1-3x), & \text{si } x \leq \frac{1}{3} \end{cases} = \begin{cases} 2 - x, & \text{si } x \geq 1 \\ 4 - 3x, & \text{si } \frac{1}{3} \leq x \leq 1 \\ 2 + 3x, & \text{si } x \leq \frac{1}{3} \end{cases}$$

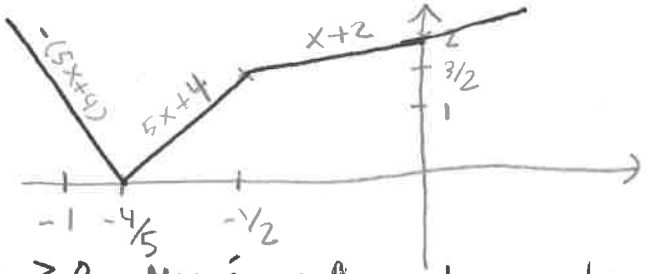


És ≥ 0 si $x \in [-2/3, 2]$.

(b) $||2x+1| - (3x+3)| \geq 0$

$$||2x+1| - (3x+3)| = \begin{cases} |(2x+1) - (3x+3)| = |-x-2| = |x+2| = x+2, & \text{si } x \geq -1/2 \\ |-(2x+1) - (3x+3)| = |-5x-4| = |5x+4|, & \text{si } x \leq -1/2 \end{cases} =$$

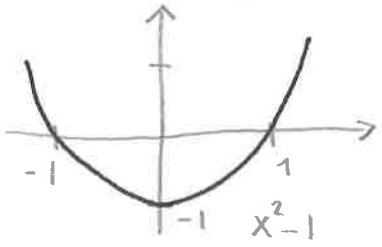
$$= \begin{cases} x+2, & \text{si } x \geq -1/2 \\ 5x+4, & \text{si } -4/5 \leq x \leq -1/2 \\ -(5x+4), & \text{si } x \leq -4/5 \end{cases}$$

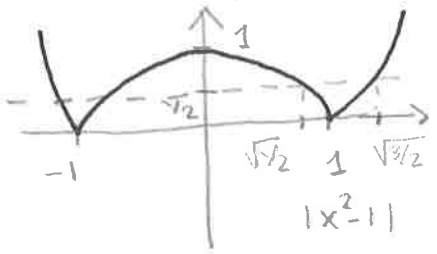


És clar que l'expressió sempre és ≥ 0. Només cal donar ubicar els seus zeros. El cjt. és $x \in \mathbb{R} \setminus \{-4/5\}$.

(c) $|x| = -x$ cert si $x \leq 0$.

(d) $|x^2 - 1| < \frac{1}{2}$





$$\left.\begin{array}{l} x^2 - 1 = \frac{1}{2} \iff x^2 = 3/2 \iff x = \pm\sqrt{3/2} \\ -(x^2-1) = +\frac{1}{2} \iff x^2 = 1/2 \iff x = \pm\sqrt{1/2} \end{array}\right\} \text{ És } (-\infty, -\sqrt{3/2}) \cup (-\sqrt{1/2}, \sqrt{1/2}) \cup (\sqrt{3/2}, +\infty)$$