(30) Proveu que $\frac{1}{9} < \sqrt{66} - 8 < \frac{1}{8}$ sense calcular $\sqrt{66}$.

Usem el teorema del valor mig amb $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$

$f(66) - f(64) = f'(c)(66-64) = \frac{1}{\sqrt{c}}$ on $c \in (64, 66)$

Per tant $64 < c < 66 < 81 \Rightarrow 8 < \sqrt{c} < 9 \Rightarrow \frac{1}{9} < \frac{1}{\sqrt{c}} < \frac{1}{8}$

Així: $\sqrt{66} - 8 = \frac{1}{\sqrt{c}} < \frac{1}{8}$; $\sqrt{66} - 8 = \frac{1}{\sqrt{c}} > \frac{1}{9}$.