(29) En els casos següents, descriviu el tipus de forma indeterminada (si n'hi ha) que s'obté per substitució directa, i avalueu el límit usant la regla de l'Hôpital, si cal.

(a) $\lim_{x \to +\infty} x \ln x = +\infty.$

(b) $\lim_{x \to 0^+} x^{1/x} = 0^{+\infty} = \lim_{x \to 0^+} e^{\frac{\ln x}{x}} = e^{\frac{-\infty}{0^+}} = e^{-\infty} = 0$

$x^{1/x} = (e^{\ln x})^{1/x} = e^{\frac{\ln x}{x}}$

(c) $\lim_{x \to 0^+} (e^x + x)^{2/x} = 1^{+\infty} = \lim_{x \to 0^+} e^{\frac{2\ln(e^x+x)}{x}} = e^4.$

on $\lim_{x \to 0^+} \frac{\ln(e^x+x)}{x} = \frac{0}{0} = \lim_{x \to 0^+} \frac{\frac{e^x+1}{e^x+x}}{1} = 2$

(d) $\lim_{x \to +\infty} x^{1/x} = +\infty^0 = \lim_{x \to +\infty} e^{\frac{\ln x}{x}} = e^0 = 1$

on $\lim_{x \to +\infty} \frac{\ln x}{x} = \frac{\infty}{\infty} = \lim_{x \to +\infty} \frac{1/x}{1} = 0.$

(e) $\lim_{x \to 4^+} [3(x-4)]^{x-4} = 0^0 = \lim_{x \to 4^+} e^{(x-4)\ln(3(x-4))} = e^0 = 1.$

on $\lim_{x \to 4^+} (x-4)\ln(3(x-4)) = 0 \cdot \infty = \lim_{x \to 4^+} \frac{\ln(3(x-4))}{1/(x-4)} = \frac{\infty}{\infty} =$

$= \lim_{x \to 4^+} \frac{\frac{3}{3(x-4)}}{-\frac{1}{(x-4)^2}} = \lim_{x \to 4^+} -(x-4) = 0$

(f) $\lim_{x \to 1^+} (\ln x)^{x-1} = 0^0 = \lim_{x \to 1^+} e^{(x-1)\ln(\ln x)} = e^0 = 1.$

on $\lim_{x \to 1^+} (x-1)\ln(\ln x) = 0 \cdot \infty = \lim_{x \to 1^+} \frac{\ln(\ln x)}{1/(x-1)} = \frac{\infty}{\infty} =$

$= \lim_{x \to 1^+} \frac{\frac{1}{\ln(x)} \cdot \frac{1}{x}}{-\frac{1}{(x-1)^2}} = -\lim_{x \to 1^+} \frac{(x-1)^2}{\ln(x)} = \frac{0}{0} = -\lim_{x \to 1^+} \frac{2(x-1)}{1/x} = 0$

(g) $\lim\limits_{x\to 0^+}\left[\cos\left(\frac{\pi}{2}-x\right)\right]^x = 0^0 = \lim\limits_{x\to 0^+} e^{x\ln\left(\cos\left(\frac{\pi}{2}-x\right)\right)} = e^0 = 1$

om $\lim\limits_{x\to 0^+} x\ln\left(\cos\left(\frac{\pi}{2}-x\right)\right) = 0\cdot\infty = \lim\limits_{x\to 0^+}\dfrac{\ln\left(\cos\left(\pi/2-x\right)\right)}{1/x} = \dfrac{\infty}{\infty} =$

$= \lim\limits_{x\to 0^+}\dfrac{\frac{-\sin(\pi/2-x)(-1)}{\cos(\pi/2-x)}}{-1/x^2} = \lim\limits_{x\to 0^+} -\dfrac{x^2\sin(\pi/2-x)}{\cos(\pi/2-x)} = -\lim\limits_{x\to 0^+}\dfrac{x^2}{\cos(\pi/2-x)} = \dfrac{0}{0} =$

$= -\lim\limits_{x\to 0^+}\dfrac{2x}{-\sin(\pi/2-x)} = 0$

(h) $\lim\limits_{x\to 2^+}\left(\dfrac{1}{x^2-4} - \dfrac{\sqrt{x-1}}{x^2-4}\right) = +\infty - \infty = \lim\limits_{x\to 2^+}\dfrac{1-\sqrt{x-1}}{(x^2-4)} = \dfrac{0}{0}$

$= \lim\limits_{x\to 2^+}\dfrac{-\frac{1}{2\sqrt{x-1}}}{2x} = -\dfrac{1/2}{4} = -1/8$

(i) $\lim\limits_{x\to 1^+}\left(\dfrac{3}{\ln x} - \dfrac{2}{x-1}\right) = +\infty - \infty = \lim\limits_{x\to 1^+}\dfrac{3(x-1)-2\ln x}{(x-1)\ln x} \underset{\uparrow}{=} \dfrac{0}{0} =$

$= \lim\limits_{x\to 1^+}\dfrac{3-2/x}{\ln x + \frac{x-1}{x}} = \dfrac{3-2}{0^+} = +\infty.$

(j) $\lim\limits_{x\to 0}\dfrac{\sqrt{25-x^2}-5}{x} = \dfrac{0}{0} = \lim\limits_{x\to 0}\dfrac{\frac{-2x}{2\sqrt{25-x^2}}}{1} = \dfrac{-2\cdot 0}{2\cdot\sqrt{25}} = 0$

(k) $\lim\limits_{x\to 0}\dfrac{e^x-(1-x)}{x} = \dfrac{0}{0} = \lim\limits_{x\to 0} e^x+1 = e^0+1 = 2.$

(l) $\lim\limits_{x\to 1}\dfrac{\ln(x^2)}{x^2-1} = \dfrac{0}{0} = \lim\limits_{x\to 1}\dfrac{2\ln x}{x^2-1} = \lim\limits_{x\to 1}\dfrac{2/x}{2x} = 1$

(m) $\lim\limits_{x\to 1}\dfrac{x^a-1}{x^b-1} \underset{\uparrow}{=} \dfrac{0}{0} = \lim\limits_{x\to 1}\dfrac{ax^{a-1}}{bx^{b-1}} = \lim\limits_{x\to 1}\dfrac{a}{b}x^{a-b} = \dfrac{a}{b}.$

$(a,b\neq 0)$

(n) $\lim\limits_{x\to 0}\dfrac{\sin(ax)}{\sin(bx)} \underset{\uparrow}{=} \dfrac{0}{0} = \lim\limits_{x\to 0}\dfrac{a\cos(ax)}{b\cos(bx)} = \dfrac{a}{b}.$

$(a,b\neq 0)$

(o) $\lim_{x\to 1} \frac{\arctan(x) - \pi/4}{x-1} = \frac{0}{0} = \lim_{x\to 1} \frac{\frac{1}{1+x^2}}{1} = \frac{1}{2}.$

(p) $\lim_{x\to\infty} \frac{\cos x}{x} = 0$ ($\cos x$ està acotada quan $x \to \infty$).

(q) $\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)} = \frac{0}{0} = \lim_{x\to 1} \frac{1/x}{\pi\cos(\pi x)} = -\frac{1}{\pi}$

(r) $\lim_{x\to 0} \frac{\arctan(x)}{\sin(x)} = \lim_{x\to 0} \frac{\frac{1}{1+x^2}}{\cos(x)} = 1.$