(23) Representem esquemàticament les següents funcions i indiquem en quins intervals $[a,b]$ són injectives.

(a) $f(x) = x^3 - 3x^2$

f és injectiva en tot interval on sigui o bé estrictament creixent o bé estrictament decreixent.

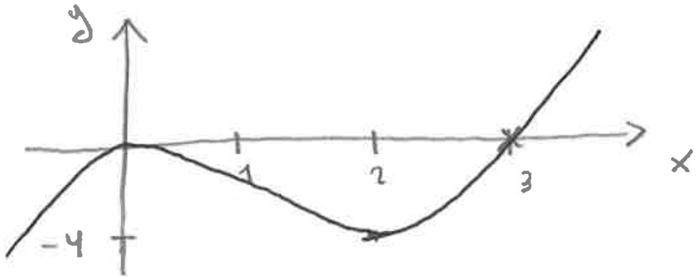
$f'(x) = 3x^2 - 6x = 3x(x-2)$

$f'(x) = 0 \iff x \in \{0, 2\}$

$f'>0$ ↗ 0 $f'<0$ ↘ 2 $f'>0$ ↗

$\lim_{x\to-\infty} f = -\infty$, $\lim_{x\to+\infty} f = +\infty$, $f(x) = x^2(x-3) = 0 \iff x \in \{0,3\}$ ($x=0$ és un zero doble)

$f(2) = -4$



f és injectiva en:

$(-\infty, 0]$

$[0, 2]$

$[2, +\infty)$

(b) $f(x) = \dfrac{x+1}{x^2+1}$ (observem $x^2+1 \neq 0$ $\forall x \in \mathbb{R}$)

$$f'(x) = \frac{x^2+1-(x+1)2x}{(x^2+1)^2} = \frac{1-2x-x^2}{(x^2+1)^2}$$

$$f'(x) = 0 \iff 1-2x-x^2 = 0 \iff x = \frac{2 \pm \sqrt{4+4}}{-2} = \frac{2 \pm 2\sqrt{2}}{-2} = -1 \mp \sqrt{2}$$

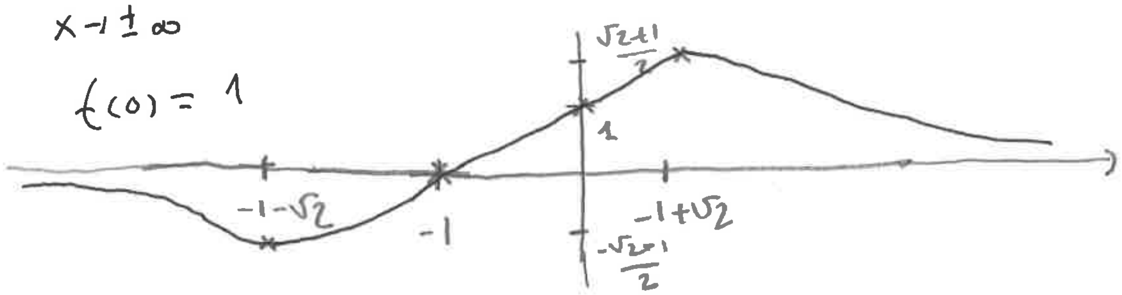
$f'<0$ ↘ $-1-\sqrt{2}$ $f'>0$ ↗ $-1+\sqrt{2}$ $f'<0$ ↘

$$f(-1-\sqrt{2}) = \frac{-\sqrt{2}}{2(2+\sqrt{2})} = \frac{-\sqrt{2}(2-\sqrt{2})}{2(4-2)} = -\frac{\sqrt{2}-1}{2}$$

$$f(-1+\sqrt{2}) = \frac{\sqrt{2}}{2(2-\sqrt{2})} = \frac{\sqrt{2}(2+\sqrt{2})}{2(4-2)} = \frac{\sqrt{2}+1}{2}$$

$\lim_{x\to\pm\infty} f = 0$, $f(x) = 0 \iff x = -1$.

$f(0) = 1$



f és injectiva en: $(-\infty, -1-\sqrt{2}]$, $[-1-\sqrt{2}, -1+\sqrt{2}]$, $[-1+\sqrt{2}, +\infty)$