(20) Utilitzant el teorema del valor mig de forma reiterada, proveu que $\tan x > x + x^3/3$, $0 < x < \pi/2$.

- $f(x) = \tan x - x - x^3/3$ i hem de veure $f(x) > 0$ si $0 < x < \pi/2$. Observem $\tan x > 0$ si $0 < x < \pi/2$ i $f(0)=0$.

$f(x) = f(x) - f(0) \underset{\substack{\uparrow \\ \text{t. valor mig}}}{=} f'(c)\cdot(x-0) = f'(c)\cdot \underbrace{x}_{>0}$ on $c \in (0, \pi/2)$,

$f'(x) = 1 + \tan^2 x - 1 - x^2 = \tan^2 x - x^2 = \underbrace{(\tan x + x)}_{>0}(\tan x - x)$

Així, si veiem $g(x) = \tan x - x$ compleix $g(x) > 0$ si $x \in (0, \pi/2)$ ja està.

$g(x) = g(x) - g(0) = g'(d)\cdot(x-0) = g'(d)\cdot \underbrace{x}_{>0}$ on $d \in (0, \pi/2)$

$g'(x) = 1 + \tan^2 x - 1 = \tan^2 x > 0 \Rightarrow g'(d) > 0.$