Exercise Sheet 6
Multivariate Calculus

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Friday, May 7th, 2021

The exercises must be handed in on Blackboard on Monday, May 24th at the latest.

Problem 1 (4 points)

Compute the volume of the solid inside of the hemisphere

$$H = \left\{(x, y, z) \in \mathbb{R}^3 \mid z \geq 0, z \leq \sqrt{16 - x^2 - y^2}\right\}$$

and outside of the cylinder given by the equation $x^2 + y^2 = 1$.

Problem 2 (4 points)

Let S be a slice of the sphere of center $(0, 0, 0)$ and radius 2, given by

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 4,\ y \geq 0, y \leq x\}.$$

Compute the integral

$$\iiint_S (x + y) dx dy dz.$$

Problem 3 (4 points)

Let R be the region of $\mathbb{R}^2$ limited by the curves $xy = 1$, $xy = 9$, $y = x$ and $y = 4x$. Compute the area of R using the change of variables

$$\begin{array}{l} u = \sqrt{xy} \\ v = \sqrt{\frac{y}{x}} \end{array} .$$

Problem 4 (4 points)

Find the line integral of the function $f(x, y, z) = x + y + z$ along the curve $\gamma(t) = \left(t, t^2, \frac{2}{3}t^3\right)$, $0 \leq t \leq 1$.

Problem 5 (4 points)

Find the line integral of the vector field $F(x, y, z) = (yz, xz, xy)$ along the curve $\gamma(t) = \left(t, t^2, t^3\right)$, for $-1 \leq t \leq 3$.