Exercise Sheet 4
Multivariate Calculus

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The exercises must be handed in on Blackboard on Monday, April 5th at the latest.

1 Problem 1 (2 points)

In each case, check if the given function satisfies the corresponding partial differential equation:

1. *(1 point)* $f(x,y) = e^x \sin y$, $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ (Laplace equation).
2. *(1 point)* $f(x,t) = \sin(x - ct)$, $\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$ (Wave equation).

2 Problem 2 (4 points)

Find and classify all the critical points of the function

$$f(x,y) = 3\log x + \log y + \log(2 - x - y).$$

3 Problem 3 (8 points)

Let $f \in \mathcal{C}^1(\mathbb{R},\mathbb{R})$ a strictly increasing function such that $f'(x) > 0$ for all $x \in \mathbb{R}$. Let $G(x,y) = (f(x), -y + xf(x))$.

1. *(2 points)* Prove that G satisfies the hypothesis of the inverse function theorem for each $(x,y) \in \mathbb{R}^2$.
2. *(2 points)* Given $(u_0, v_0) \in \mathrm{Im}(G)$, compute $DG^{-1}(u_0, v_0)$.
3. *(2 points)* Prove that G is an injective function.
4. *(2 points)* Compute G^{-1} explicitly.

4 Problem 4 (6 points)

A radio telescope is to be installed in a point on the surface of a spherical planet. The location needs to be selected in a way that the interference with the magnetic field is minimal. If the radius of the planet is $R = 5$ and the strength of the vector field is $g(x,y,z) = x^2 - y^2 + z^2 - 2x - 4z + 9$ (considering the origin of the coordinate system to be in the center of the planet), determine the best point(s) in which to install the telescope.