

Exercise Sheet 3

Multivariate Calculus

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Friday, March 12th, 2021

The exercises must be handed in on Friday, March 19th at the latest.

1 Problem 1 (2 points)

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ a function with the property that $|f(x)| \leq \|x\|^2$. Show that f is differentiable at $(0,0)$.

2 Problem 2 (9 points)

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by

$$f(x, y) = \begin{cases} \frac{x \sin(y^2)}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}.$$

Study the following questions:

- (a) (1 point) Is f continuous at $(0,0)$?
- (b) (3 points) Does $\frac{\partial f}{\partial x}$ exist at $(0,0)$? Is $\frac{\partial f}{\partial x}$ continuous at $(0,0)$?
- (c) (3 points) Does $\frac{\partial f}{\partial y}$ exist at $(0,0)$? Is $\frac{\partial f}{\partial y}$ continuous at $(0,0)$?
- (d) (2 points) Is f differentiable at $(0,0)$?

3 Problem 3 (9 points)

Study the same questions for the function

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}.$$