

Exercise Sheet 2

Multivariate Calculus

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Friday, February 26th, 2021

The exercises must be handed in on Friday, March 5th at the latest.

1 Recursive sequence (4 points)

Let $(x_n)_n$ the sequence defined by

$$\begin{aligned}x_0 &= \sqrt{2} \\ x_{n+1} &= \sqrt{2 + x_n} \ .\end{aligned}$$

Show that it converges and compute its limit.

2 Convergence and absolute convergence (3 points)

Is the series $\sum_{k=1}^{\infty} (-1)^k \frac{\log k}{k^2}$ convergent? Is it absolutely convergent?

3 Series convergence with one parameter (5 points)

Study the convergence of the following series with respect to the parameter $\alpha \in \mathbb{Z}$:

- $\sum_{k=1}^{\infty} \frac{1}{1 + 2^\alpha + \dots + k^\alpha}$
- $\sum_{k=1}^{\infty} \left(1 + \frac{1}{2^\alpha} + \dots + \frac{1}{k^\alpha} \right)$

4 Series convergence with two parameters (8 points)

For which values of $\alpha, \beta \in \mathbb{R}$ do the following series converge?

- $\sum_{k=1}^{\infty} \frac{1}{k^\alpha (\log k)^\beta}$
- $\sum_{k=1}^{\infty} k^\alpha e^{-\beta k}$