Nordhaus-Gaddum-type results for locating domination [1]

Ignacio M. Pelayo[2]

[2]Department de Matemàtica Aplicada III
Universitat Politècnica de Catalunya
Barcelona, Catalunya, Spain

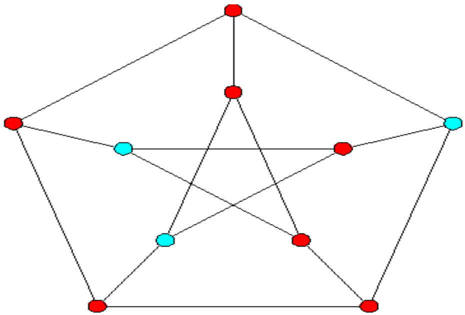
CanaDAM 2013
MUN, St. John's, NL, Canada

[1]Joint work with Carmen Hernando and Mercè Mora.

▷ A set $D \subset V(G)$ of a graph G is a *dominating set* if every vertex of $V \setminus D$ has a neighbour in D, i.e., for each $u \in V \setminus D$, $N(u) \cap D \neq \emptyset$.

- ▷ A set $D \subset V(G)$ of a graph G is a *dominating set* if every vertex of $V \setminus D$ has a neighbour in D, i.e., for each $u \in V \setminus D$, $N(u) \cap D \neq \emptyset$.
- ▷ The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.

- ▷ A set $D \subset V(G)$ of a graph G is a *dominating set* if every vertex of $V \setminus D$ has a neighbour in D, i.e., for each $u \in V \setminus D$, $N(u) \cap D \neq \emptyset$.
- ▷ The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.

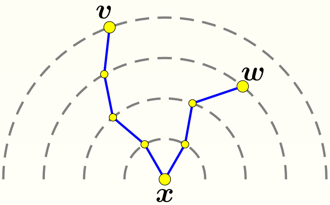


- $\gamma(P) = 3$, since blue vertices form a minimum dominating set.

Let $G = (V, E)$ be a connected graph and $v, w \in V$.

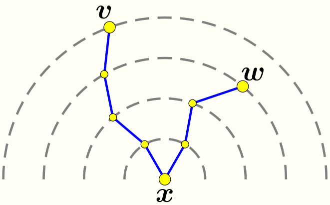
Let $G = (V, E)$ be a connected graph and $v, w \in V$.

▷ A vertex $x \in V$ *resolves* the pair $\{v, w\}$ if $d(x, v) \neq d(x, w)$



Let $G = (V, E)$ be a connected graph and $v, w \in V$.

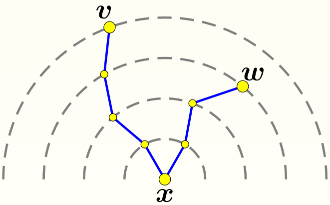
- ▷ A vertex $x \in V$ *resolves* the pair $\{v, w\}$ if $d(x, v) \neq d(x, w)$



- ▷ A set $S \subseteq V$ is a *locating set* of G if every pair $v, w \in V$ are resolved by some vertex $x \in S$.

Let $G = (V, E)$ be a connected graph and $v, w \in V$.

▷ A vertex $x \in V$ *resolves* the pair $\{v, w\}$ if $d(x, v) \neq d(x, w)$



▷ A set $S \subseteq V$ is a *locating set* of G if every pair $v, w \in V$ are resolved by some vertex $x \in S$.

▷ Let $S = \{u_1, \ldots, u_k\}$ be a locating set. The ordered set:

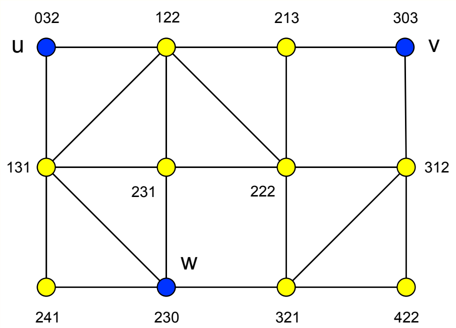
$$[d(x, u_1), \ldots, d(x, u_k)]$$

is the vector of *metric coordinates* of $x \in V$ with respect to S.

▷ *Metric basis* of G: locating set of minimum cardinality.

▷ *Metric basis* of G: locating set of minimum cardinality.

▷ *Metric dimension* of G, $\beta(G)$: cardinality of a metric basis.



- In this graph, $\{u, v, w\}$ is a metric basis.

▷ A set D of vertices in a graph G is a *locating dominating set* if it is both locating and dominating.

- A set D of vertices in a graph G is a *locating dominating set* if it is both locating and dominating.

- The *metric-location-domination number* $\eta(G)$ is the minimum cardinality of a locating dominating set of G.

- ▷ A set D of vertices in a graph G is a *locating dominating set* if it is both locating and dominating.

- ▷ The *metric-location-domination number* $\eta(G)$ is the minimum cardinality of a locating dominating set of G.

$\Longrightarrow$ Let $S_1, S_2 \subseteq V(G)$. If S_1 is dominating and S_2 is locating, then $S_1 \cup S_2$ is both locating and dominating. Hence,

- ▷ A set D of vertices in a graph G is a *locating dominating set* if it is both locating and dominating.

- ▷ The *metric-location-domination number* $\eta(G)$ is the minimum cardinality of a locating dominating set of G.

$\Longrightarrow$ Let $S_1, S_2 \subseteq V(G)$. If S_1 is dominating and S_2 is locating, then $S_1 \cup S_2$ is both locating and dominating. Hence,

$$\max\{\gamma(G), \beta(G)\} \leq \eta(G) \leq \gamma(G) + \beta(G)$$

- A set D of vertices in a graph G is a *locating-dominating set*, or simply an *LD-set*, if for every two vertices $u, v \in V(G) \setminus D$,

$$\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset.$$

- A set D of vertices in a graph G is a *locating-dominating set*, or simply an *LD-set*, if for every two vertices $u, v \in V(G) \setminus D$,

$$\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset.$$

- The *location-domination number* $\lambda(G)$ is the minimum cardinality of an LD-set of G.

- A set D of vertices in a graph G is a *locating-dominating set*, or simply an *LD-set*, if for every two vertices $u, v \in V(G) \setminus D$,

$$\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset.$$

- The *location-domination number* $\lambda(G)$ is the minimum cardinality of an LD-set of G.
- A λ*-code* is an LD-set of cardinality $\lambda(G)$.

- A set D of vertices in a graph G is a *locating-dominating set*, or simply an *LD-set*, if for every two vertices $u, v \in V(G) \setminus D$,

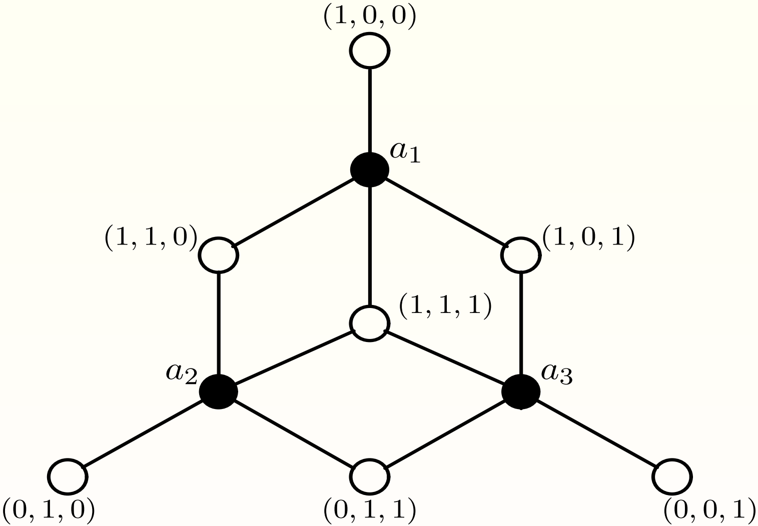
$$\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset.$$

- The *location-domination number* $\lambda(G)$ is the minimum cardinality of an LD-set of G.
- A λ*-code* is an LD-set of cardinality $\lambda(G)$.

$\Longrightarrow$ Every locating-dominating set is both locating and dominating. Hence,

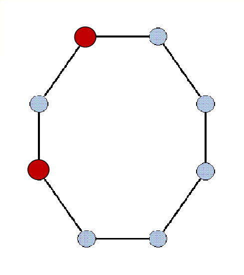
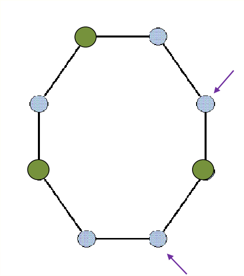
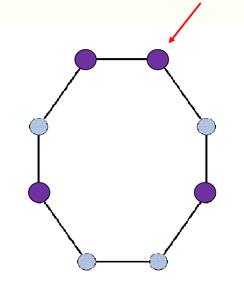
$$\max\{\gamma(G), \beta(G)\} \leq \eta(G) \leq \min\{\lambda(G), \gamma(G) + \beta(G)\}$$

and both bounds are tight.



In all cases, digit 0 means "greater than 1"

$\lambda(G) = 3$, since $\{a_1, a_2, a_3\}$ is a λ-code.

		
$\beta(G) = 2$	$\gamma(G) = \eta(G) = 3$	$\lambda(G) = 4$

In this example:

$$\max\{\gamma(G), \beta(G)\} = 3 \leq \eta(G) = 3 \leq \min\{\lambda(G), \gamma(G) + \beta(G)\} = 4$$

$G = (V, E)$ is a graph and $\overline{G} = (V, \overline{E})$ is its complementary graph.

$G = (V, E)$ is a graph and $\overline{G} = (V, \overline{E})$ is its complementary graph.

▷ A dominating set $D \subseteq V$ of G is a *global dominating set* if it is also a dominating set of $\overline{G}$.

$G = (V, E)$ is a graph and $\overline{G} = (V, \overline{E})$ is its complementary graph.

- A dominating set $D \subseteq V$ of G is a *global dominating set* if it is also a dominating set of $\overline{G}$.
- An LD-set S of G is called *global* if it is a global dominating set.

$G = (V, E)$ is a graph and $\overline{G} = (V, \overline{E})$ is its complementary graph.

- ▷ A dominating set $D \subseteq V$ of G is a *global dominating set* if it is also a dominating set of $\overline{G}$.
- ▷ An LD-set S of G is called *global* if it is a global dominating set.

$\Rightarrow$ An LD-set S is global iff no vertex $u \in V$ satisfies $S \subseteq N_G(u)$.

$G = (V, E)$ is a graph and $\overline{G} = (V, \overline{E})$ is its complementary graph.

- ▷ A dominating set $D \subseteq V$ of G is a *global dominating set* if it is also a dominating set of $\overline{G}$.
- ▷ An LD-set S of G is called *global* if it is a global dominating set.

$\Rightarrow$ An LD-set S is global iff no vertex $u \in V$ satisfies $S \subseteq N_G(u)$.

$\Rightarrow$ A set S is a global LD-set of G iff it is a global LD-set of $\overline{G}$.

$G=(V,E)$ is a graph and $\overline{G}=(V,\overline{E})$ is its complementary graph.

- ▷ A dominating set $D\subseteq V$ of G is a *global dominating set* if it is also a dominating set of $\overline{G}$.
- ▷ An LD-set S of G is called *global* if it is a global dominating set.
- ⇒ An LD-set S is global iff no vertex $u\in V$ satisfies $S\subseteq N_G(u)$.
- ⇒ A set S is a global LD-set of G iff it is a global LD-set of $\overline{G}$.
- ▷ Equivalently, an LD-set S of G is called *non-global* if there exists a vertex $w\in V\setminus S$ such that $S\subseteq N(w)$. The vertex w, which is necessarily unique, is called the *dominating vertex* of S.

$G=(V,E)$ is a graph and $\overline{G}=(V,\overline{E})$ is its complementary graph.

- ▷ A dominating set $D \subseteq V$ of G is a *global dominating set* if it is also a dominating set of $\overline{G}$.
- ▷ An LD-set S of G is called *global* if it is a global dominating set.
- ⇒ An LD-set S is global iff no vertex $u \in V$ satisfies $S \subseteq N_G(u)$.
- ⇒ A set S is a global LD-set of G iff it is a global LD-set of $\overline{G}$.
- ▷ Equivalently, an LD-set S of G is called *non-global* if there exists a vertex $w \in V \setminus S$ such that $S \subseteq N(w)$. The vertex w, which is necessarily unique, is called the *dominating vertex* of S.
- ⇒ If S is a non-global LD-set of G, then $S+w$ is an LD-set of $\overline{G}$.

$\Rightarrow$ If G contains a global λ-code, then $\lambda(\overline{G}) \leq \lambda(G)$.

⇒ If G contains a global λ-code, then $\lambda(\overline{G}) \leq \lambda(G)$.

⇒ If every λ-code of G is non-global , then $\lambda(\overline{G}) \leq \lambda(G) + 1$.

$\Rightarrow$ If G contains a global λ-code, then $\lambda(\overline{G}) \leq \lambda(G)$.

$\Rightarrow$ If every λ-code of G is non-global , then $\lambda(\overline{G}) \leq \lambda(G) + 1$.

$$\Longrightarrow |\lambda(G) - \lambda(\overline{G})| \leq 1$$

$\Rightarrow$ If G contains a global λ-code, then $\lambda(\overline{G}) \leq \lambda(G)$.

$\Rightarrow$ If every λ-code of G is non-global , then $\lambda(\overline{G}) \leq \lambda(G) + 1$.

$$\Longrightarrow |\lambda(G) - \lambda(\overline{G})| \leq 1$$

$\Rightarrow 2\lambda(G) - 1 \leq \lambda(G) + \lambda(\overline{G}) \leq 2\lambda(G) + 1$

$\Rightarrow$ If G contains a global λ-code, then $\lambda(\overline{G}) \leq \lambda(G)$.

$\Rightarrow$ If every λ-code of G is non-global , then $\lambda(\overline{G}) \leq \lambda(G) + 1$.

$$\Longrightarrow |\lambda(G) - \lambda(\overline{G})| \leq 1$$

$$\Rightarrow 2\lambda(G) - 1 \leq \lambda(G) + \lambda(\overline{G}) \leq 2\lambda(G) + 1$$

▷ G is of type I: $\boxed{\lambda(G) - 1 \leq \lambda(\overline{G}) \leq \lambda(G)}$

$\Rightarrow$ If G contains a global λ-code, then $\lambda(\overline{G}) \leq \lambda(G)$.

$\Rightarrow$ If every λ-code of G is non-global , then $\lambda(\overline{G}) \leq \lambda(G) + 1$.

$$\Longrightarrow |\lambda(G) - \lambda(\overline{G})| \leq 1$$

$$\Rightarrow 2\lambda(G) - 1 \leq \lambda(G) + \lambda(\overline{G}) \leq 2\lambda(G) + 1$$

▷ G is of type I: $\boxed{\lambda(G) - 1 \leq \lambda(\overline{G}) \leq \lambda(G)}$

▷ G is of type II: $\boxed{\lambda(\overline{G}) = \lambda(G) + 1}$

- Let S be a non-global λ-code of a graph $G = (V, E)$.

- Let S be a non-global λ-code of a graph $G = (V, E)$.
- Let w be the dominating vertex of S, i.e., s.t. $S \subseteq N_G(w)$. Then,

- Let S be a non-global λ-code of a graph $G = (V, E)$.
- Let w be the dominating vertex of S, i.e., s.t. $S \subseteq N_G(w)$. Then,

  $\Rightarrow$ $ecc(w) \leq 2$.

- Let S be a non-global λ-code of a graph $G = (V, E)$.
- Let w be the dominating vertex of S, i.e., s.t. $S \subseteq N_G(w)$. Then,

  $\Rightarrow$ $ecc(w) \leq 2$.

  $\Rightarrow$ G is connected.

- Let S be a non-global λ-code of a graph $G = (V, E)$.
- Let w be the dominating vertex of S, i.e., s.t. $S \subseteq N_G(w)$. Then,

  $\Rightarrow$ $ecc(w) \leq 2$.

  $\Rightarrow$ G is connected.

  $\Rightarrow$ $rad(G) \leq 2$.

- Let S be a non-global λ-code of a graph $G = (V, E)$.
- Let w be the dominating vertex of S, i.e., s.t. $S \subseteq N_G(w)$. Then,

  $\Rightarrow$ $ecc(w) \leq 2$.

  $\Rightarrow$ G is connected.

  $\Rightarrow$ $rad(G) \leq 2$.

  $\Rightarrow$ $diam(G) \leq 4$.

- Let S be a non-global λ-code of a graph $G = (V, E)$.
- Let w be the dominating vertex of S, i.e., s.t. $S \subseteq N_G(w)$. Then,

  $\Rightarrow$ $ecc(w) \leq 2$.

  $\Rightarrow$ G is connected.

  $\Rightarrow$ $rad(G) \leq 2$.

  $\Rightarrow$ $diam(G) \leq 4$.

  $\Rightarrow$ $\Delta(G) \geq \lambda(G)$.

$\Longrightarrow$ If G is a graph satisfying at least one the following conditions, then $\lambda(\overline{G}) \leq \lambda(G)$.

$\Longrightarrow$ If G is a graph satisfying at least one the following conditions, then $\lambda(\overline{G}) \leq \lambda(G)$.

- $\star$ G is disconnected.

$\Longrightarrow$ If G is a graph satisfying at least one the following conditions, then $\lambda(\overline{G}) \leq \lambda(G)$.

- $\star$ G is disconnected.
- $\star$ $rad(G) \geq 3$.

$\Longrightarrow$ If G is a graph satisfying at least one the following conditions, then $\lambda(\overline{G}) \leq \lambda(G)$.

- $\star$ G is disconnected.
- $\star$ $rad(G) \geq 3$.
- $\star$ $diam(G) \geq 5$.

$\Longrightarrow$ If G is a graph satisfying at least one the following conditions, then $\lambda(\overline{G}) \leq \lambda(G)$.

- ⋆ G is disconnected.
- ⋆ $rad(G) \geq 3$.
- ⋆ $diam(G) \geq 5$.
- ⋆ $\Delta(G) < \lambda(G)$.

$\Longrightarrow$ If G is a graph satisfying at least one the following conditions, then $\lambda(\overline{G}) \leq \lambda(G)$.

- $\star$ G is disconnected.
- $\star$ $rad(G) \geq 3$.
- $\star$ $diam(G) \geq 5$.
- $\star$ $\Delta(G) < \lambda(G)$.

- Moreover, all conditions are tight.

$\Longrightarrow$ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

$\Longrightarrow$ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

$\star$ If T is a tree, then the following statements are equivalent:

- $diam(T) = 2$.
- $T \cong K_{1,n-1}$ (i.e., T is a star).
- $\overline{T}$ is disconnected
- $\lambda(\overline{T}) = \lambda(T) = n - 1$.

$\Longrightarrow$ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

$\star$ If T is a tree, then the following statements are equivalent:

- $diam(T) = 2$.
- $T \cong K_{1,n-1}$ (i.e., T is a star).
- $\overline{T}$ is disconnected
- $\lambda(\overline{T}) = \lambda(T) = n - 1$.

$\star$ If T is a tree other than P_4,, then the following statements are equivalent:

- $diam(T) = 3$.
- $T \cong K_2(r, s)$ (i.e., T is a double star).
- $\lambda(\overline{T}) = \lambda(T) - 1 = n - 2$.

$\Longrightarrow$ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

⋆ If T is a tree, then the following statements are equivalent:

- $diam(T) = 2$.
- $T \cong K_{1,n-1}$ (i.e., T is a star).
- $\overline{T}$ is disconnected
- $\lambda(\overline{T}) = \lambda(T) = n - 1$.

⋆ If T is a tree other than P_4,, then the following statements are equivalent:

- $diam(T) = 3$.
- $T \cong K_2(r, s)$ (i.e., T is a double star).
- $\lambda(\overline{T}) = \lambda(T) - 1 = n - 2$.

⋆ $\lambda(\overline{P_n}) = \lambda(P_n) - 1$ if $n \in \{5k + 1, 5k + 3\}$, otherwise $\lambda(\overline{P}_n) = \lambda(P_n)$.

$\Longrightarrow$ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

⋆ If T is a tree, then the following statements are equivalent:

- $diam(T) = 2$.
- $T \cong K_{1,n-1}$ (i.e., T is a star).
- $\overline{T}$ is disconnected
- $\lambda(\overline{T}) = \lambda(T) = n - 1$.

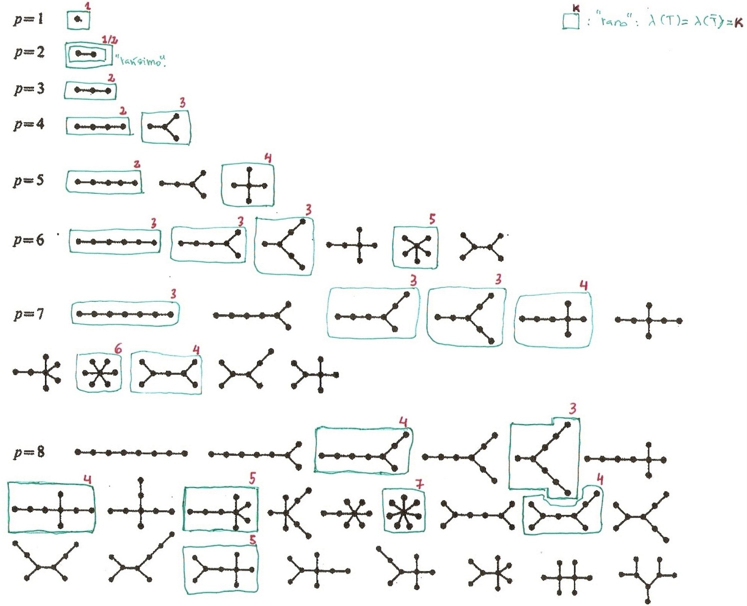
⋆ If T is a tree other than P_4,, then the following statements are equivalent:

- $diam(T) = 3$.
- $T \cong K_2(r, s)$ (i.e., T is a double star).
- $\lambda(\overline{T}) = \lambda(T) - 1 = n - 2$.

⋆ $\lambda(\overline{P_n}) = \lambda(P_n) - 1$ if $n \in \{5k + 1, 5k + 3\}$, otherwise $\lambda(\overline{P_n}) = \lambda(P_n)$.

⋆ $\lambda(\overline{P_{n/2,n/2}}) = \lambda(P_{n/2,n/2}) - 1$, $n \neq 6$, $\lambda(\overline{P_{3,3}}) = \lambda(P_{3,3})$.

There are 48 trees of order at most 8, 23 of them s.t. $\lambda(\overline{T}) = \lambda(T)$.



$$\Rightarrow \lambda(\overline{K}_n) = n = \lambda(K_n) + 1.$$

$\Rightarrow$ $\lambda(\overline{K}_n) = n = \lambda(K_n) + 1$.

$\Rightarrow$ Let G be a non-complete graph. If $\lambda(\overline{G}) = \lambda(G) + 1$, then

$\Rightarrow$ $\lambda(\overline{K}_n) = n = \lambda(K_n) + 1$.

$\Rightarrow$ Let G be a non-complete graph. If $\lambda(\overline{G}) = \lambda(G) + 1$, then

- ⋆ Every λ-code of G is non-global.

$\Rightarrow$ $\lambda(\overline{K}_n) = n = \lambda(K_n) + 1$.

$\Rightarrow$ Let G be a non-complete graph. If $\lambda(\overline{G}) = \lambda(G) + 1$, then

- ⋆ Every λ-code of G is non-global.
- ⋆ $rad(G) \leq 2$ and $diam(G) \leq 4$.

$\Rightarrow$ $\lambda(\overline{K}_n) = n = \lambda(K_n) + 1$.

$\Rightarrow$ Let G be a non-complete graph. If $\lambda(\overline{G}) = \lambda(G) + 1$, then

- $\star$ Every λ-code of G is non-global.
- $\star$ $rad(G) \leq 2$ and $diam(G) \leq 4$.

$\star$ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

$\Rightarrow$ $\lambda(\overline{K}_n) = n = \lambda(K_n) + 1$.

$\Rightarrow$ Let G be a non-complete graph. If $\lambda(\overline{G}) = \lambda(G) + 1$, then

- ⋆ Every λ-code of G is non-global.
- ⋆ $rad(G) \leq 2$ and $diam(G) \leq 4$.

⋆ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

- What about other cactus graphs?

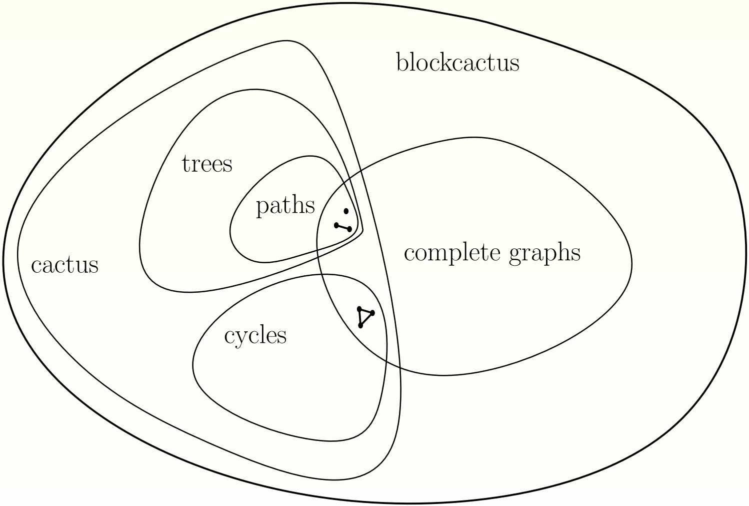
$\Rightarrow$ $\lambda(\overline{K}_n) = n = \lambda(K_n) + 1$.

$\Rightarrow$ Let G be a non-complete graph. If $\lambda(\overline{G}) = \lambda(G) + 1$, then

- ⋆ Every λ-code of G is non-global.
- ⋆ $rad(G) \leq 2$ and $diam(G) \leq 4$.

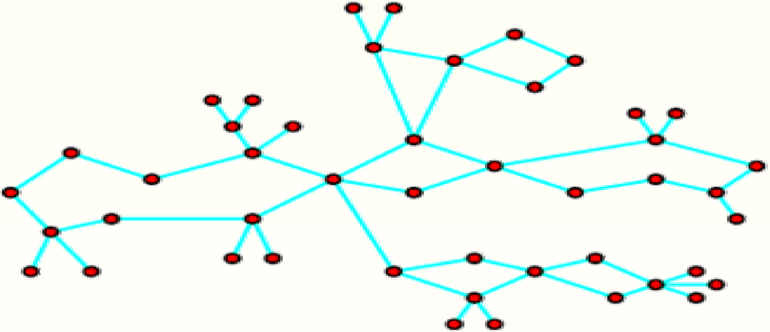
⋆ If T is a tree other than K_2, then $\lambda(\overline{T}) \leq \lambda(T)$.

- What about other cactus graphs?
- What about other block-cactus graphs?


blockcactus
trees
paths
cactus
complete graphs
cycles

A *cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a single edge.

A *cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a single edge.



A *cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a single edge.

- $G = (V, E)$ is a cactus graph having a non-global λ-code S.

A *cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a single edge.

- $G = (V, E)$ is a cactus graph having a non-global λ-code S.
- Let u be the dominating vertex of S.

A *cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a single edge.

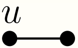
- $G = (V, E)$ is a cactus graph having a non-global λ-code S.
- Let u be the dominating vertex of S.
- Let $\{V_1\}_{i=1}^{r}$ be the sets of vertices of the components of $G - u$.

$\Longrightarrow$ Then, for every integer $i \in \{1, \ldots, r\}$, $G_i = G[V_i + u]$ is isomorphic to some of the graphs displayed below.

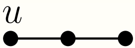
A *cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a single edge.

- $G = (V, E)$ is a cactus graph having a non-global λ-code S.
- Let u be the dominating vertex of S.
- Let $\{V_1\}_{i=1}^{r}$ be the sets of vertices of the components of $G - u$.

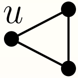
$\Longrightarrow$ Then, for every integer $i \in \{1, \ldots, r\}$, $G_i = G[V_i + u]$ is isomorphic to some of the graphs displayed below.



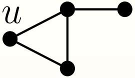
(1) K_2



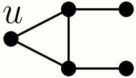
(2) P_3



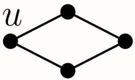
(3) C_3



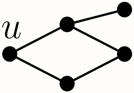
(4) paw



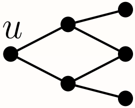
(5) bull



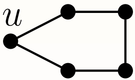
(6) C_4



(7) banner



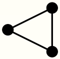
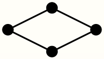
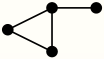
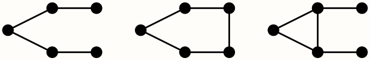
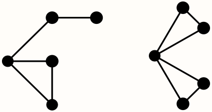
(8) corner



(9) C_5

- Case $\lambda = 2$:

- Case $\lambda = 2$:

	$\lambda(\overline{G}) = \lambda(G) = 2$	$\lambda(\overline{G}) = \lambda(G) + 1 = 3$
$n = 3$		
$n = 4$	 	
$n = 5$		

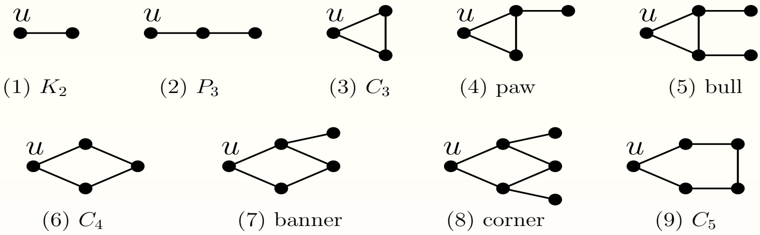
- Case $\lambda > 2$:

- Case $\lambda > 2$:

$\Rightarrow$ Let u be the dominating vertex of a non-global λ-code S of G. Let $\{V_1\}_{i=1}^{r}$ be the sets of vertices of the components of $G - u$. The subgraphs $\{G_i = G[V_i + u]\}_{i=1}^{r}$ are isomorphic to:

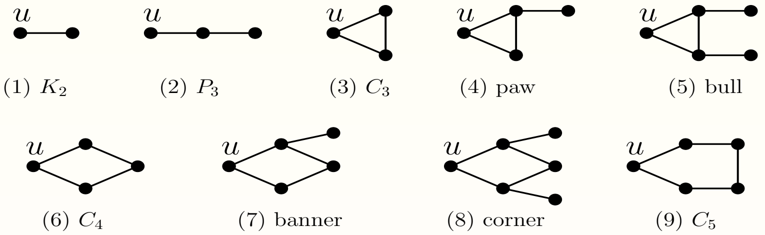
- Case $\lambda > 2$:

$\Rightarrow$ Let u be the dominating vertex of a non-global λ-code S of G.
Let $\{V_1\}_{i=1}^{r}$ be the sets of vertices of the components of $G - u$.
The subgraphs $\{G_i = G[V_i + u]\}_{i=1}^{r}$ are isomorphic to:



- Case $\lambda > 2$:

$\Rightarrow$ Let u be the dominating vertex of a non-global λ-code S of G. Let $\{V_1\}_{i=1}^{r}$ be the sets of vertices of the components of $G - u$. The subgraphs $\{G_i = G[V_i + u]\}_{i=1}^{r}$ are isomorphic to:

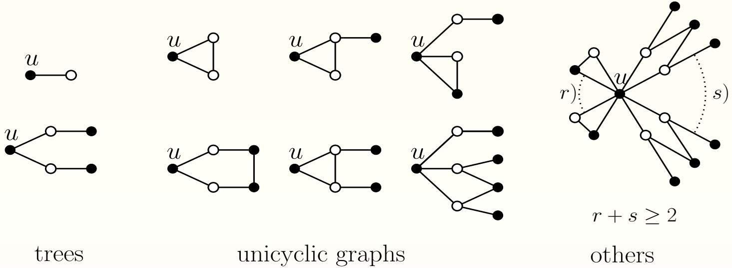


$\Longrightarrow$ Then, G contains a global λ-code if either

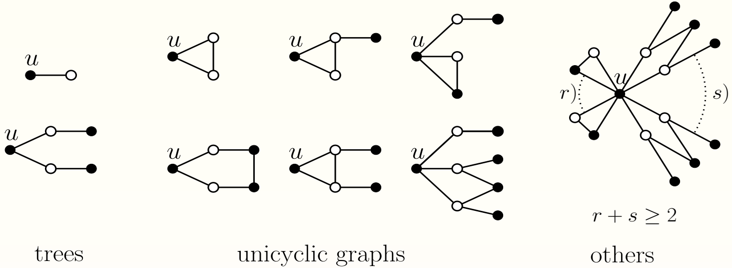
1. For some $i \in \{1, \ldots, r\}$, $G_i \in \{(1), (4), (5), (6), (7), (9)\}$, or
2. $r \geq 3$ and for some $i \in \{1, \ldots, r\}$, G_i is isomorphic to (2).

$\Rightarrow$ A cactus graph does not contain any global λ-code if an only if it is isomorphic to one of the graphs showed below.

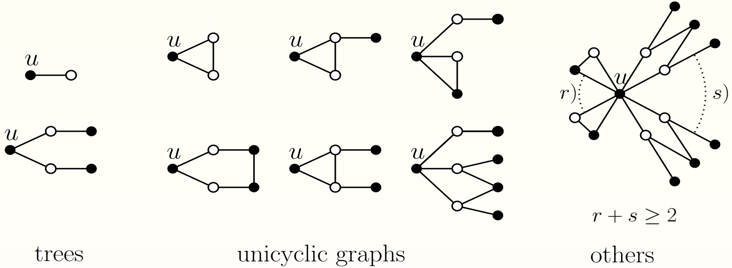
⇒ A cactus graph does not contain any global λ-code if an only if it is isomorphic to one of the graphs showed below.

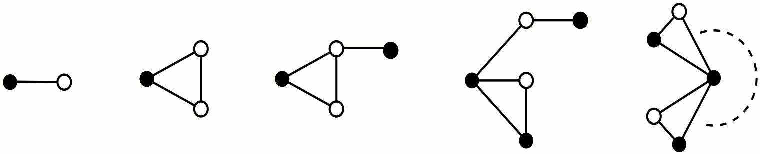


⇒ A cactus graph does not contain any global λ-code if an only if it is isomorphic to one of the graphs showed below.



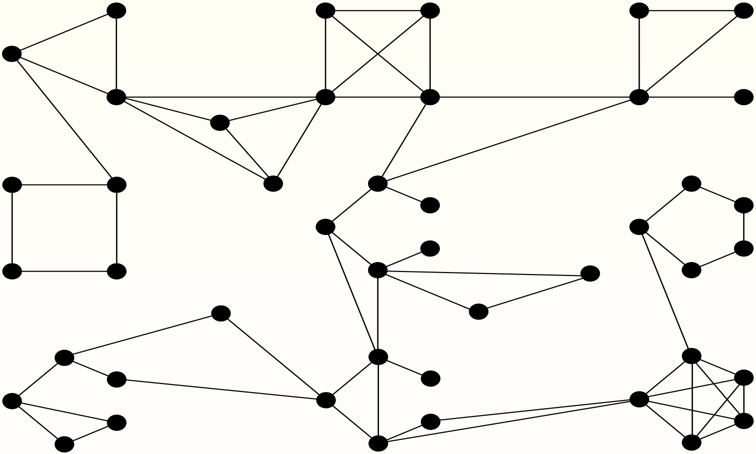
⇒ A cactus graph does not contain any global λ-code if an only if it is isomorphic to one of the graphs showed below.





A *block-cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a clique.

A *block-cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a clique.

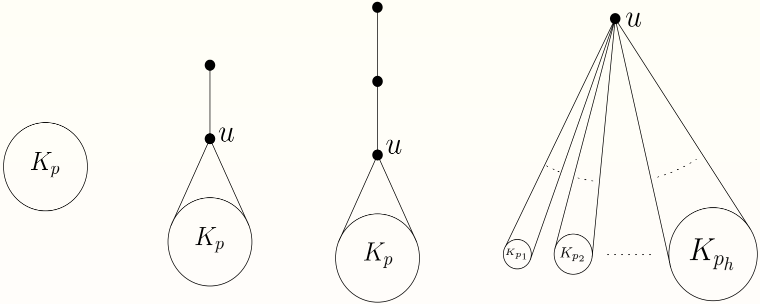


A *block-cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a clique.

⇒ Let G be a block-cactus graph. Then, $\lambda(\overline{G}) = \lambda(G) + 1$ if an only if it is isomorphic to some of the graphs displayed below.

A *block-cactus graph* is a connected graph in which every block (maximal subgraph without a cut-vertex) is either a cycle or a clique.

⇒ Let G be a block-cactus graph. Then, $\lambda(\overline{G}) = \lambda(G) + 1$ if an only if it is isomorphic to some of the graphs displayed below.



- K_p denotes a complete graph of order $p \geq 2$.

Solving $\lambda(\overline{G}) = \lambda(G) + 1$ for Bipartite Graphs

Solving $\lambda(\overline{G}) = \lambda(G) + 1$ for Bipartite Graphs

$\Rightarrow$ If $G \cong K_2(r, s)$, then $\lambda(\overline{G}) = \lambda(G) - 1$.

Solving $\lambda(\overline{G}) = \lambda(G) + 1$ for Bipartite Graphs

⇒ If $G \cong K_2(r, s)$, then $\lambda(\overline{G}) = \lambda(G) - 1$.

⇒ If $G \cong K_{r,s}$, then $\lambda(\overline{G}) = \lambda(G)$.

Solving $\lambda(\overline{G}) = \lambda(G) + 1$ for Bipartite Graphs

$\Rightarrow$ If $G \cong K_2(r, s)$, then $\lambda(\overline{G}) = \lambda(G) - 1$.

$\Rightarrow$ If $G \cong K_{r,s}$, then $\lambda(\overline{G}) = \lambda(G)$.

- $G = (V = U \cup W, E)$ is a bipartite graph of order $n = r + s$.
- The order of G is $n = r + s$, with $2 \leq |U| = r \leq |W| = s$.

Solving $\lambda(\overline{G}) = \lambda(G) + 1$ for Bipartite Graphs

$\Rightarrow$ If $G \cong K_2(r, s)$, then $\lambda(\overline{G}) = \lambda(G) - 1$.

$\Rightarrow$ If $G \cong K_{r,s}$, then $\lambda(\overline{G}) = \lambda(G)$.

- $G = (V = U \cup W, E)$ is a bipartite graph of order $n = r + s$.
- The order of G is $n = r + s$, with $2 \leq |U| = r \leq |W| = s$.

$\Longrightarrow$ If $\lambda(\overline{G}) = \lambda(G) + 1$, then

- $\star$ $r \geq 3$.
- $\star$ If $r = 3$, then $s \in \{5, 6\}$.
- $\star$ If $r \geq 4$, then $s \leq 2^r - 1$.
- $\star$ If $r \geq 4$, then $2r + 1 \leq s$ [Conjectured, but not yet proved].