On the superconnectivity of generalized p-cycles

C. Balbuena, I. Pelayo*
Departament de Matemàtica Aplicada III

J. Gómez†
Departament de Matemàtica Aplicada i Telemàtica

Universitat Politècnica de Catalunya

Abstract

A generalized p-cycle is a digraph whose set of vertices can be partitioned into p parts that are cyclically ordered in such a way that the vertices in one part are adjacent only to vertices in the next part. Any digraph can be shown as a p-cycle with $p = 1$, and bipartite digraphs are generalized p-cycles with $p = 2$. A maximally connected digraph is said to be superconnected if every disconnecting set of δ vertices or edges is trivial, where δ stands for the minimum degree. In this work we study the problem of disconnecting p-cycles by removing nontrivial subsets of vertices or edges. To be more precise, after obtaining optimal lower bounds for the parameters κ_1, λ_1, that measure the superconnectivities, we present sufficient conditions for a p-cycle to be superconnected, and also sufficient conditions to guarantee optimum values of superconnectivities of a p-cycle. Finally, we apply our results to compute the superconnectivities of the family of De Bruijn generalized cycles.

Key words. directed graph, p-cycle, line digraph, superconnectivity, diameter, parameter ℓ.

AMS subject classification. 05C40, 05C20

References

[1] M. Aigner, On the linegraph of a directed graph. *Math. Z.* **102** (1967), 56–61.

[2] F.T Boesch, Synthesis of reliable networks–A survey, *IEEE Trans. Reliability* **35** (1986), 240-246.

*e-mail: m.camino.balbuena@upc.es ignacio.m.pelayo@upc.es
†e-mail:jgomez@mat.upc.es

[3] C. Balbuena, A. Carmona, On the connectivity and superconnectivity of bipartite digraphs and graphs, Appearing in *Ars combinatoria.*

[4] C. Balbuena, I. Pelayo, J. Gómez, On the connectivity of generalized p-cycles, Appearing in *Ars combinatoria.*

[5] J.-C. Bermond, N. Homobono, and C. Peyrat, Large fault-tolerant interconnection networks. *Graphs and Combinatorics* **5** (1989) 107–123.

[6] F. Boesch and R. Tindell, Circulants and their connectivities, *J. Graph Theory*, **8** (1984) 487–499.

[7] M.A. Fiol, On super-edge-connected digraphs and bipartite digraphs. *J. Graph Theory* **16** (1992) 545–555.

[8] J. Fàbrega and M. A. Fiol, Maximally connected digraphs. *J. Graph Theory* **13** (1989) 657–668.

[9] J. Fàbrega and M. A. Fiol, Bipartite graphs and digraphs with maximum connectivity. *J. Discrete Applied Mathematics* **69** 3 (1996) 269–278.

[10] M.A. Fiol, J. Fàbrega and M. Escudero, Short paths and connectivity in graphs and digraphs. *Ars Combin.* **29B** (1990) 17–31.

[11] M.A. Fiol, J.L.A. Yebra and I. Alegre, Line digraph iterations and the (d, k) digraph problem. *IEEE Trans. Comput.* **C-33** (1984), 400–403.

[12] D. Geller and F. Harary, Connectivity in digraphs. *Lec. Not. Math.* **186**, Springer, Berlin (1970) 105–114.

[13] J. Gómez, C. Padró and S. Perennes, Large Generalized Cycles. *Discrete Applied Mathematics.* **89** (1998) 107–123.

[14] Y.O. Hamidoune, A property of α-fragments of a digraph. *Discrete Math.* **31** (1980) 105–106.

[15] Y.O. Hamidoune, A.S. Lladó and O. Serra, Vosperian and superconnected abelian Cayley digraphs. *Graphs and Combinatorics* **7** (1991) 143–152.

[16] M. Imase, T. Soneoka, and K. Okada, Connectivity of regular directed graphs with small diameter. *IEEE Trans. Comput.* **C-34** (1985), 267–273

[17] L. Lesniak, Results on the edge-connectivity of graphs. *Discrete Math.* **8** (1974) 351–354.

[18] S.M. Reddy, J.G. Kuhl, S.H. Hosseini and H. Lee, On digraphs with minimum diameter and maximum connectivity. *Proceedings of the 20th Annual Allerton Conference* (1982), 1018–1026.

[19] T. Soneoka, Super edge-connectivity of dense digraphs and graphs, *Discrete Appl. Math.* **37/38** (1992), 511-523.