

Some structural, metric and convex properties of the boundary of a graph*

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Abstract

Let u, v be two vertices of a connected graph G . The vertex v is said to be a *boundary vertex* of u if no neighbor of v is further away from u than v . The boundary of a graph is the set of all its boundary vertices. In this work, we present a number of properties of the boundary of a graph under different points of view: (1) a realization theorem involving different types of boundary vertex sets: extreme set, periphery, contour, and the whole boundary; (2) the contour is a monophonic set; and (3) the cardinality of the boundary is an upper bound for both the metric dimension and the determining number of a graph.

Keywords: Boundary; Contour; Extreme set; Graph convexity; Metric dimension.

1 Introduction and basic definitions

Boundary vertex sets have been studied in different contexts: *facility location*, to determine the appropriate location for certain kinds of facilities [10]; *rebuilding in graphs*, how to rebuild a graph from a certain subset of vertices by using a convex hull operator [4, 5, 7]; and *resolvability in graphs*, a

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