Some relations between the partition dimension and the twin number of a graph

Carmen Hernando
Mercè Mora
Ignacio M. Pelayo

Spain

UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH
UPC

8th Slovenian ICGT 2015

Outline

- Metric dimension: β(G)
- Partition dimension: βp(G)
- Relation β(G) & βp(G)
- Twin number τ(G)
- Relation βp(G) & τ(G) when τ(G)>n/2
- βp(G)=n-1
- βp(G)=n-2
- Realization theorem βp(G) & τ(G)

Metric Dimension

Let G = (V,E) be a graph.

Let v ϵ V, S=$\{v_1,\ldots v_r\} \subseteq$ V, we denote

$$r(v|S) = (d(v, v_1),\ldots, d(v, v_r)).$$

S is a *resolving set* of G if, for any distinct vertices u, v ϵ V, $r(u|S) \neq r(v|S)$

The *metric dimension* β(G) of G is the minimum cardinality of a resolving set of G.

Partition Dimension

$G = (V;E)$

$S \subseteq V$, $d(v, S) = \min\{d(v,w) : w \in S\}$

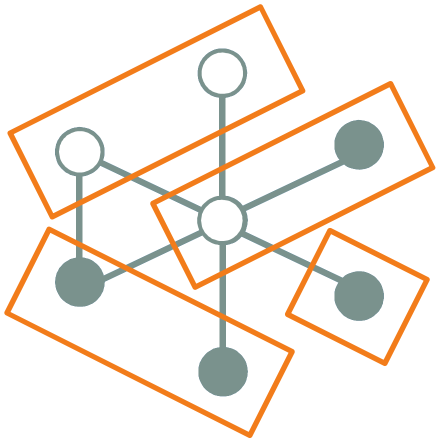
If $\pi = \{S_1,\ldots,S_k\}$ is a partition of V, we denote

$$r(v|\pi) = (d(v, S_1),\ldots, d(v, S_k)).$$

π is a *resolving partition* of G if, for any distinct vertices $u, v \in V$, $r(u|\pi) \neq r(v|\pi)$

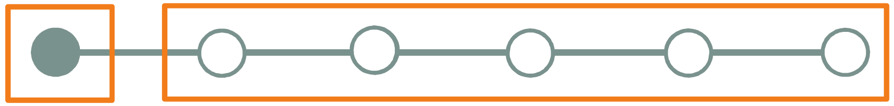
The *partition dimension* $\beta p(G)$ of G is the minimum cardinality of a resolving partition of G.

Metric Dimension & Partition Dimension



β(G)=4
β_P(G)=4

β(G)=1 β_P(G)=2



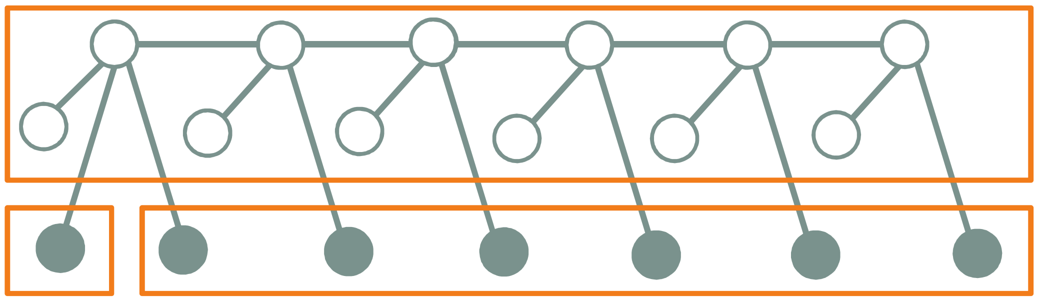
If S=$\{v_1,\ldots,v_r\}$ is a resolvig set of G then π=$\{\{v_1\},\ldots,\{v_r\}$, V\S} is a resolving partition of G

βp(G)≤ β(G)+1

[ChSZ'2000]

[ChGH'2005]

if 3 ≤α≤ β+1 then ∃G: βp(G)= α, β(G)= β



β(G)=7
β_P(G)=3

C. Hernando
M. Mora, I. Pelayo

Partition Dimension

References

[ChSZ'2000] G. Chartrand, E. Salehi and P. Zhang,
The partition dimension of a graph,
Aequationes Mathematicae, 59, 45--54.

[T'2008] I. Tomescu,
Discrepancies between metric dimension and partition dimension of a connected graph,
Discrete Mathematics, 308, 5026--5031.

Partition Dimension

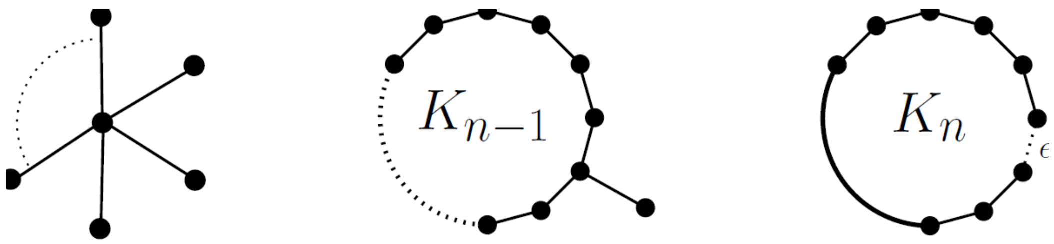
[ChSZ'2000]

$\beta p(G) = 1 \iff G = P_1$

$\beta p(G) = 2 \iff G = P_n$, $n \geq 2$.

$\beta p(G) = n \iff G = K_n$

$\beta p(G) = n-1 \iff G = K_{1,n-1}$ or $K_1 \vee (K_1 + K_{n-2})$ or $G = K_n - e$.



(a) $K_{1,n-1}$ (b) $K_1 \vee (K1 + K_{n-2})$ (c) $K_n - e$

$\beta p(G) = n-2 \iff G \in \mathcal{G}_1 \cup \cdots \cup \mathcal{G}_{23}$ **[T'2008]**

Other Results About Partition Dimension

Study in some families of graphs:

- •Trees [Rodríguez-Velázquez, González Yero, Lemanska'2014]
- •Circulant Graphs [Grigorious, Stephen, Sudeep, Rajan, Miller, William'2014]
- •Wheel related graphs [Javaid, Shokat'2008]
- •Infinite regular graphs [Tomescu, Imran'2009]
- •Cayley digraphs [Fehr, Gosselin, Oellermann'2006]

•Study of behaviour with product graphs operations

- •Corona product [Baskoro,Darmaji'2012]
- •Cartesian product [González Yero, Rodríguez-Velázquez'2010]
- •Strong product [González Yero,Jakovac,Kuziak,Taranenko'2014]

•Other variations

- •Connected partition dimension [Saenpholphat, Zhang'2002]
- •Path partition dimension [Verman'2007]
- •Star partition dimension [Marinescu-Ghemeci'2012]
- •Fault-tolerant p. dimension [Chaudhry, Javaid, Salman'2010]
- •Strong partition dimension [González Yero'2013+]

Twin Number

u, v are *twins* $\Leftrightarrow$ $N(u)\setminus\{u,v\}=N(v)\setminus\{u,v\}$

u, v are twins $\Leftrightarrow$ $d(u, z) = d(v, z)\ \forall z\in V\setminus\{u,v\}$.

- The twin relation is an equivalence relation.
- Every equivalence class is either a clique or a stable set
- Twin set: equivalent class of the twin relation
- Twin set induce a complete graph or an empty graph.

The *twin number of a graph G*, denoted by $\tau(G)$, is the maximum cardinality of a twin set of G.

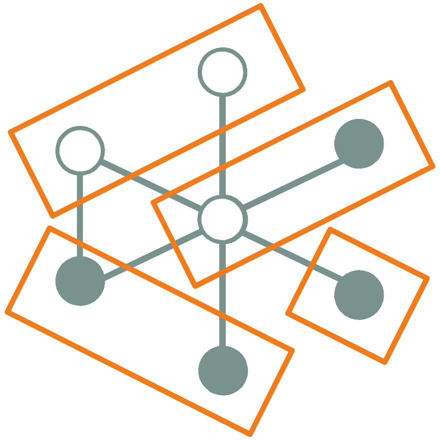
Twin Number

$\tau(G)=n \Leftrightarrow G=K_n$

$\tau(G)=n-1 \Leftrightarrow G=K_{1,n-1}$

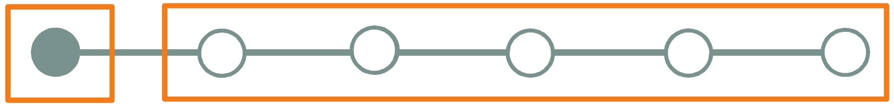
$$\tau(G)=n-2 \Leftrightarrow \begin{cases} G=K_{2,n-2} \\ G=K_2 \vee \overline{K_{n-2}}=K_{2,n-2}+e \\ G=\overline{K_2} \vee K_{n-2}=K_n-e \\ G=K_1+(K_1 \vee K_{n-2}) \end{cases}$$

Twin Number & Partition Dimension

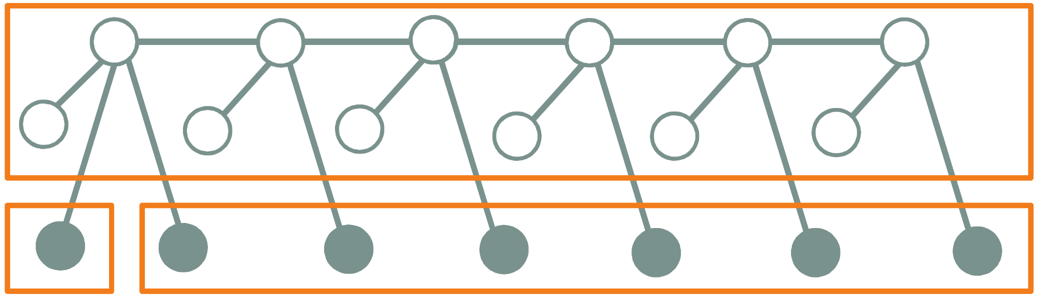


β(G)=4
β_P(G)=4
τ(G)=4

β(G)=1 β_P(G)=2 τ(G)=1 (twin free graph)



If $\pi = \{S_1,\ldots,S_k\}$ is a resolvig partition of G and B is a twin set of G then $|S_i \cap B| \leq 1$



$\tau(G) \leq \beta p(G) \leq \beta(G)+1$

β(G)=7 β_P(G)=3 τ(G)=3

C. Hernando
M. Mora, I. Pelayo

Questions

- Which graphs verify βp(G)= τ(G)?
- βp (G)=n-k $\Rightarrow$? $\leq \tau$(G) $\leq$ n-k?
- For any 1≤a ≤b, there exist graph with, τ(G)=a, βp(G)=b?

Results

I.- βp(G) versus τ(G) when τ(G) is almost its order.
II.- Bounds
III.-Caracterization of graphs with βp(G)=n-2
IV.-Realization Theorem

βp(G) versus τ(G) (τ(G)>n/2)

Let G be a connected graph of order n such that τ(G)=τ.
Let B be a twin set of cardinality τ.

Proposition 1. If τ>n/2 and G[B]=$\overline{K_\tau}$, then β_p (G) = τ.

Proposition 2 . If G[B]=K_τ, then β_p (G) $\geq \tau + 1$.

βp(G) versus τ(G) (τ(G)>n/2)

Let G be a connected graph of order n such that $\tau(G)=\tau > n/2$.
Let B be a twin set of cardinality τ.

Theorem 1.
- $\beta_p(G)=\tau$ if and only if $G[B]=\overline{K_\tau}$,
- $\beta_p(G)>\tau$ if and only if $G[B]=K_\tau$,

Proposition 3
If $G[B]=K_\tau$ and $\exists v \in V \setminus B$: $d(v,z)\neq d(v,B)$ $\forall z \in N(B)\setminus B$,
then $\beta_p(G)=\tau+1$.

C. Hernando
M. Mora, I. Pelayo

βp(G) versus τ(G) (τ(G)>n/2)

Let G be a connected graph of order n such that τ(G)= τ> n/2.

Theorem 2.
If τ(G)=n-k, then β_p(G)≤n-k/2.

Corollary 1. $\tau \leq \beta_p$(G)≤(n+ τ)/2.

Corollary 2. $2\beta_p$(G)–n $\leq \tau \leq \beta_p$(G)

Partition Dimension n-1

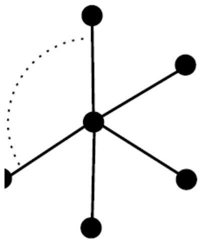
Let G=(V,E) be a connected graph of order n,

if β_p(G)=n-1 then n-1≤ τ(G)≤n-2

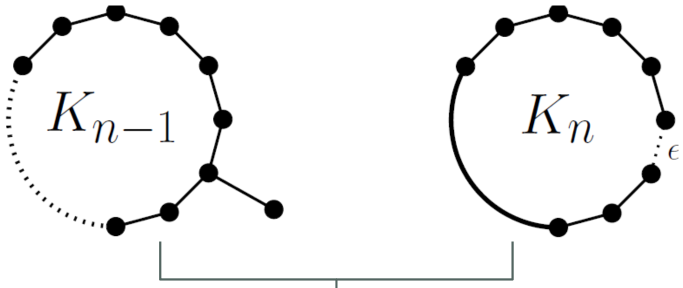
Let B be a twin set of cardinality τ.

β_p(G)=n-1 if and only if: (1) τ(G)=n-1 and G[B]=$\overline{K_\tau}$

(2) τ(G)=n-2 and G[B]=K_τ.



(1)



(2)

[ChSZ'2000]

Partition Dimension n-2

Let G=(V,E) be a connected graph of order n,

if β_p(G)=n-2 then n-2≤ τ(G)≤n-4

β_p(G)=n-2 if and only if

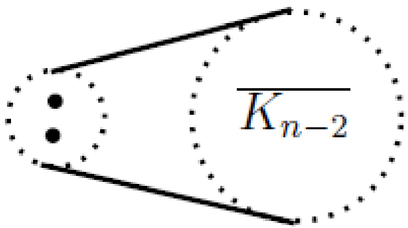
(1) τ(G)=n-2 and G[B]= $\overline{K_\tau}$ (where B is a twin set of cardinality τ)

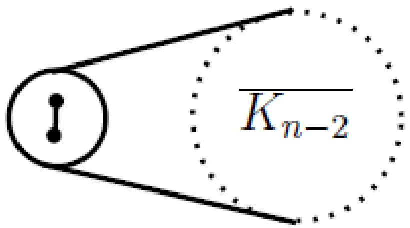
(2) τ(G)=n-3, G[B]=K_τ and ∃v ϵV\B: d(v,z)≠d(v,B) ∀z ϵN(B)\B

(3) τ(G)=n-4, G[B]=K_τ and ∄v ϵV\B: d(v,z)≠d(v,B) ∀z ϵN(B)\B

Partition Dimension n-2

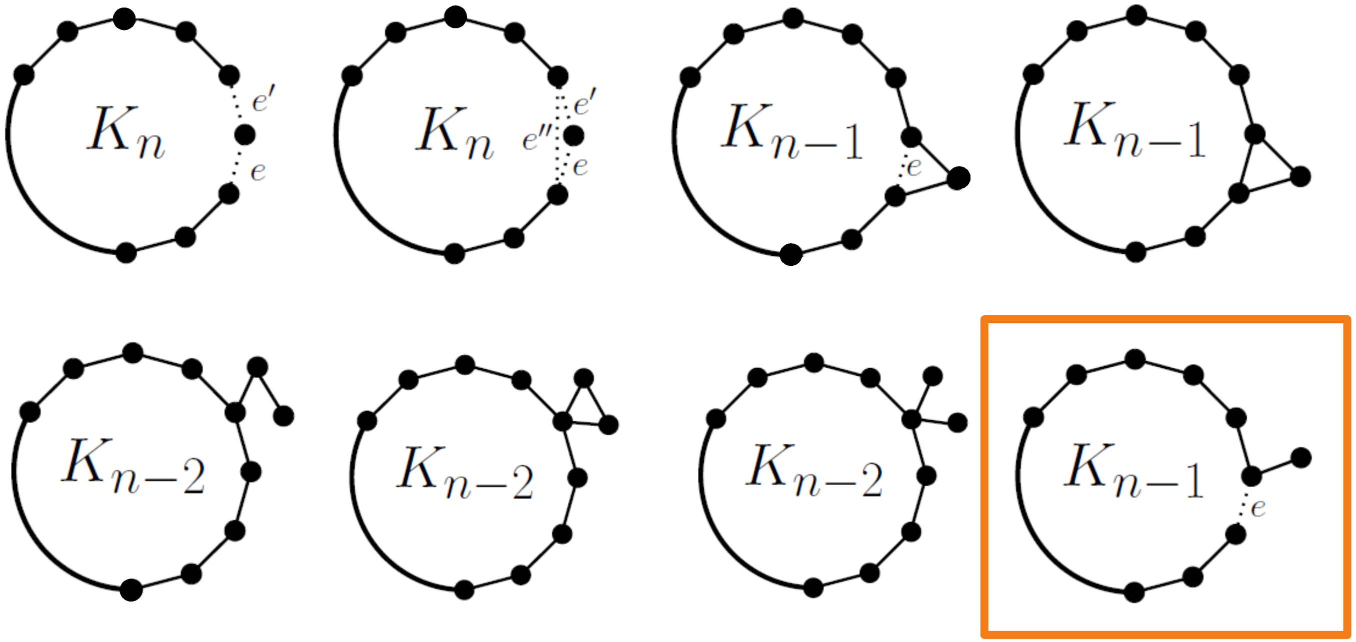
(1) τ(G)=n-2 and G[B]=$\overline{K_\tau}$ (where B is a twin set of cardinality τ)





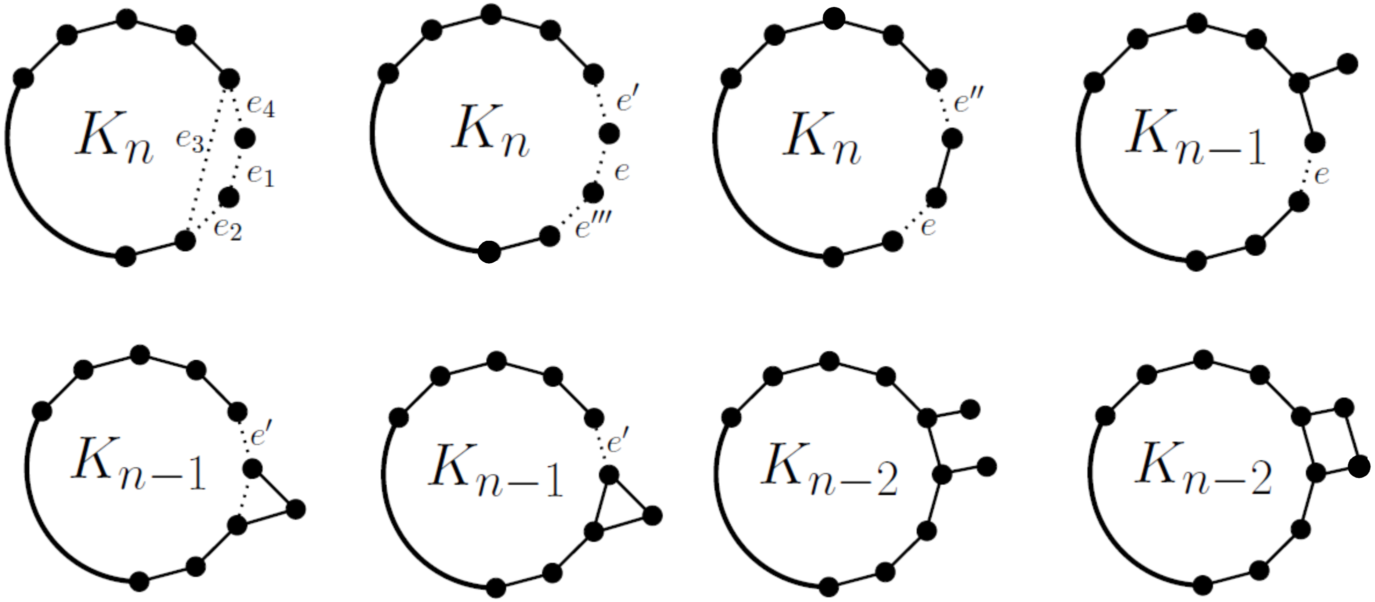
Partition Dimension n-2

(2) $\tau(G)$=n-3, G[B]=K_τ and ∃v ϵV\B: d(v,z)≠d(v,B) ∀z ϵN(B)\B



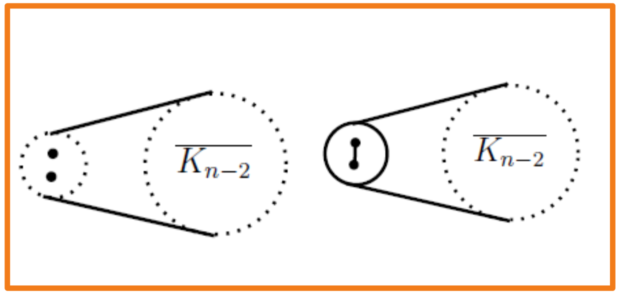
Partition Dimension n-2

(3) $\tau(G)=n-4$, $G[B]=K_\tau$ and $\nexists v \in V\setminus B$: $d(v,z)\neq d(v,B)$ $\forall z \in N(B)\setminus B$

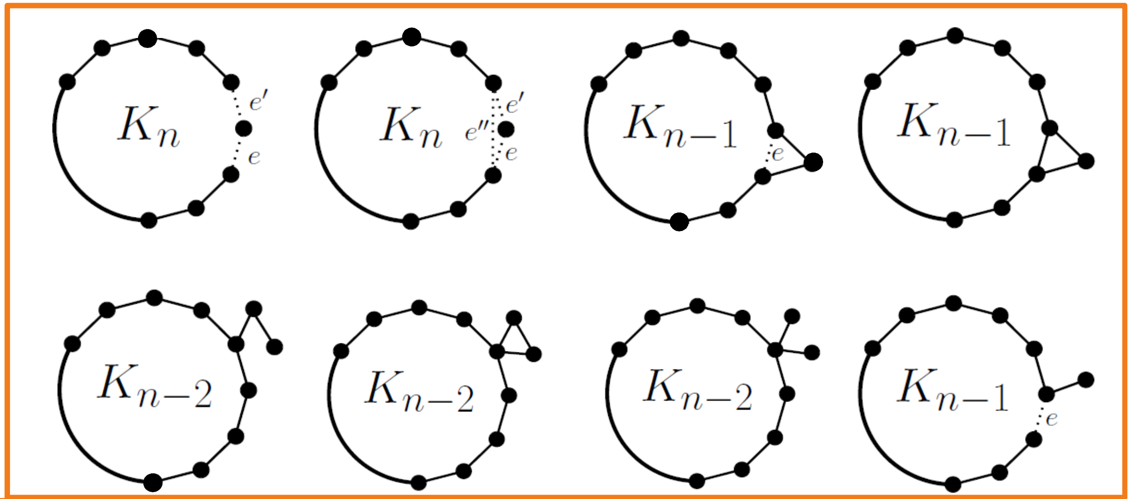


Partition Dimension n-2 [our result]

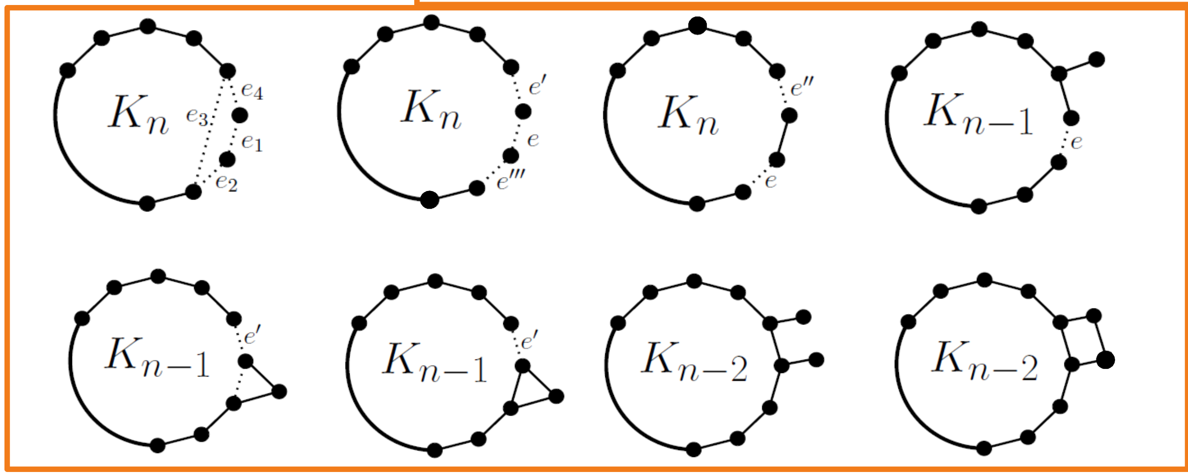
Theorem 3. $\beta p(G) = n-2 \iff G \in \mathcal{F}_1 \cup \cdots \cup \mathcal{F}_{18}$



$\tau(G)$=n-2



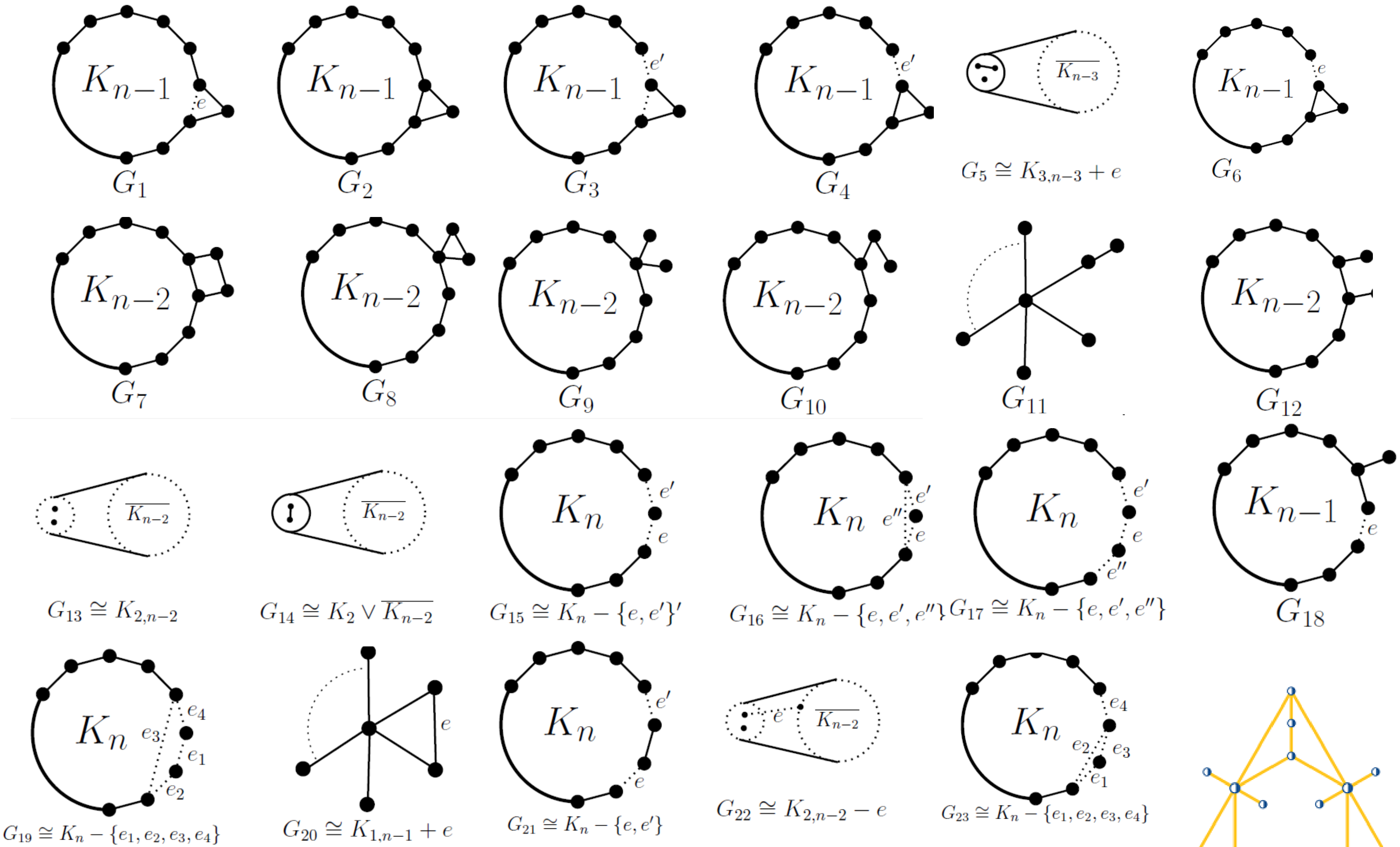
$\tau(G)$=n-3



$\tau(G)$=n-4

C. Hernando
M. Mora, I. Pelayo

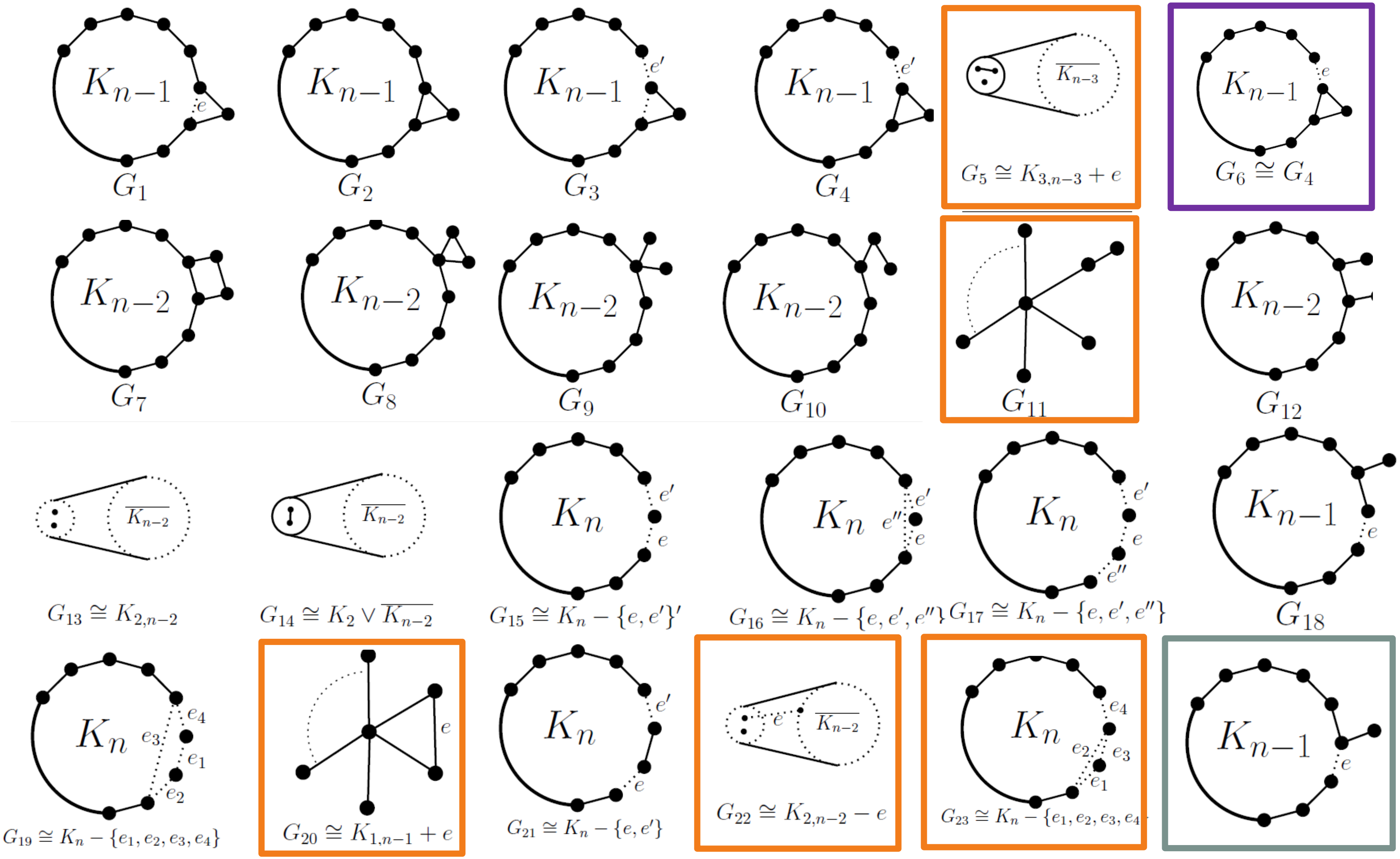
Partition Dimension n-2 [T'2008]



C. Hernando
M. Mora, I. Pelayo

Some Relations between the partition dimension and the twin number of a graph

Partition Dimension n-2



C. Hernando
M. Mora, I. Pelayo

Realization Theorem

Theorem 4.
Given a,b$\in \mathbb{N}$, $1 \leq a \leq b$ then $\exists G$: $\tau(G)=a$ and $\beta_p(G)=b$.

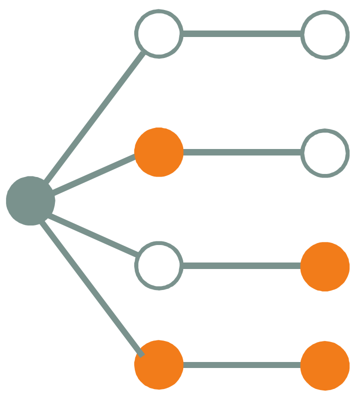
Cases:

- $1 = a = b$ G=P_1
- $2 \leq a = b$ G=$K_{1,n-1}$
- $1 = a < b$ G=G(1,b)
- $2 \leq a < b$ G=G(a,b)

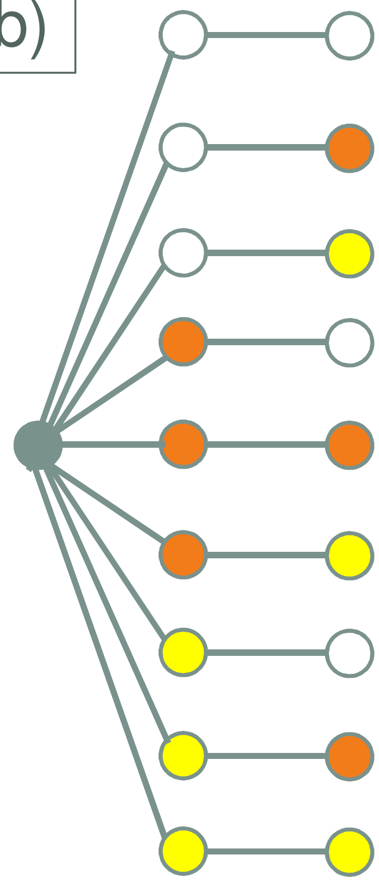
Realization Theorem

1 = a < b   G=G(1,b)

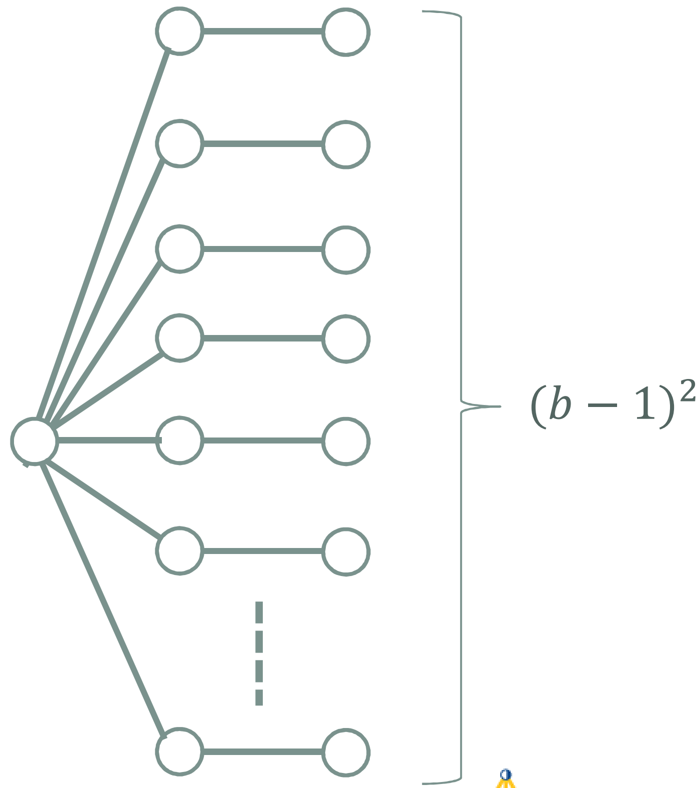
G=G(1,2)



G=G(1,3)



G=G(1,4)

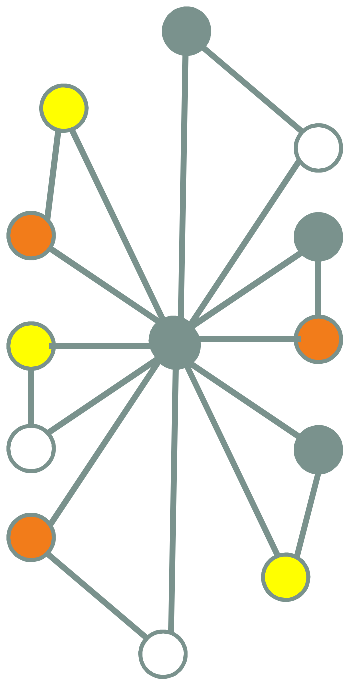


G=G(1,b)

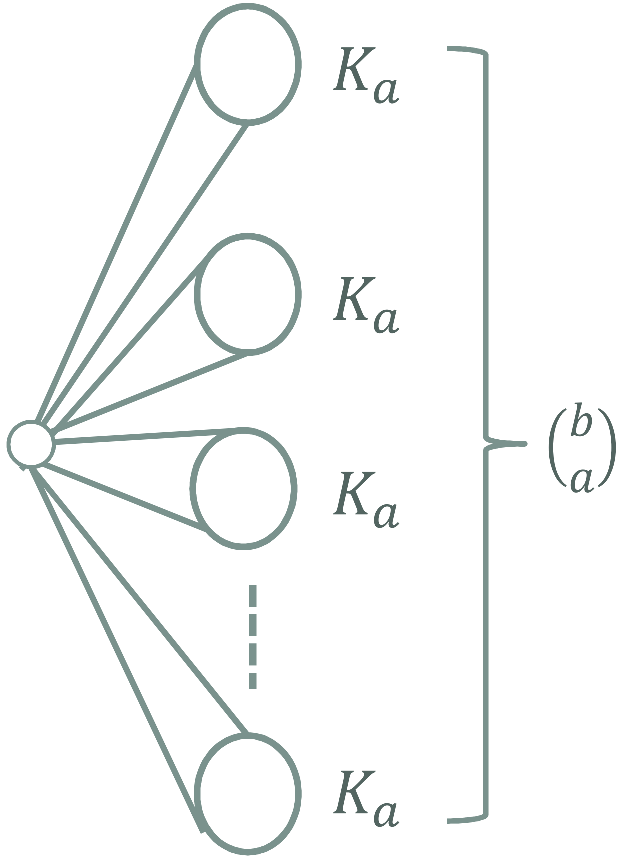
C. Hernando
M. Mora, I. Pelayo

Realization Theorem

$2 \leq a < b$ $\quad$ $G = G(a,b)$

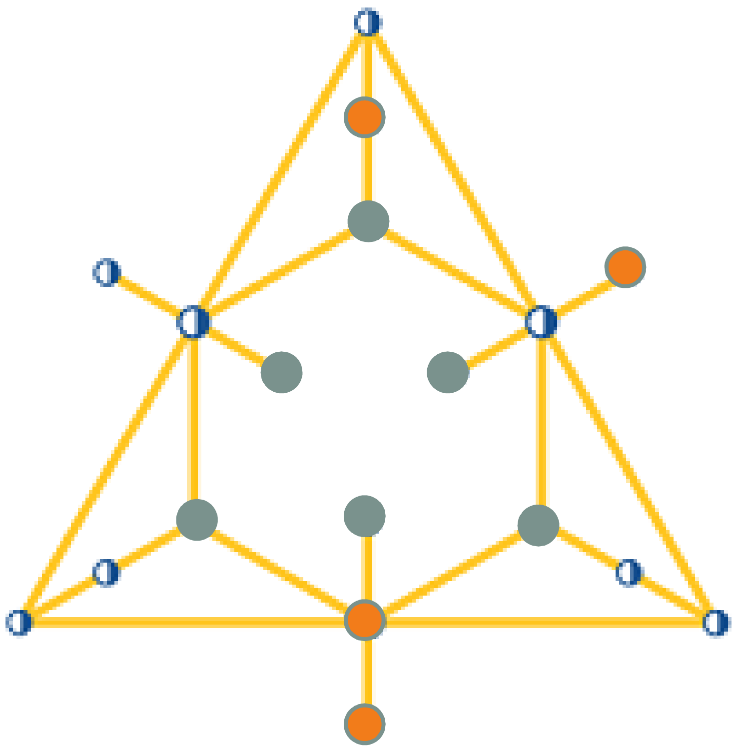


G=G(2,4)



G=G(a,b)

Thanks



Thank you for your attention!