Diameter, short paths and superconnectivity in digraphs

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Abstract

A connected digraph of minimum degree δ is said to be maximally connected if the cardinality of every cutset is al least δ, and it is called superconnected if moreover every minimum disconnecting set F consists of the vertices adjacent to or from a given vertex not belonging to F. Let π be an integer, $0 \leq \pi \leq \delta - 2$, δ being the minimum degree of the digraph. In this work, we prove that if $D \leq 2\ell^{\pi} - 2$ and $\ell^0 \geq 2$, where ℓ^{π} is a parameter related to the shortest paths, then G is maximally connected or has a good superconnectivity depending only on whether $\pi \leq \lfloor \delta/2 \rfloor$ and $\delta \geq 3$, or $\pi \leq \lfloor (\delta-2)/2 \rfloor$ and $\delta \geq 5$, respectively. In the edge case, it is enough that $D \leq 2\ell^{\pi} - 1$. Finally, the obtained results are applied to the iterated line digraphs.

Key words. Connectivity, superconnectivity, cutset, digraph, line digraph.

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