Using a progressive withdrawal procedure to study superconnectivity in ℓ^1-digraphs

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Abstract

A maximally connected digraph is said to be superconnected if every minimum disconnecting set F of vertices is trivial, i.e., it consists of the vertices adjacent to or from a given vertex not belonging to F. This work is devoted to presenting a sufficient condition — in terms of the so called parameter ℓ^1 — on the diameter, in order to guarantee that the digraph is superconnected, giving also a lower bound for the superconnectivity parameter κ_1 when non-trivial disconnecting sets exist. This result has been achieved with the help of a 'progressive withdrawal procedure' that establishes how far away a vertex can be to or from a given set of vertices. An analogous result is presented in terms of edges, assuring edge-superconnectivity and giving a lower bound for the parameter λ_1.

Key words. Connectivity, superconnectivity, cutset, digraph.

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