On the metric dimension of graph products

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A vertex x of a graph G is said to resolve two vertices u and v of G if $d(x,u) \neq d(x,v)$. An ordered vertex set S of a graph G is a *resolving set* of G if every two distinct vertices of G are resolved by some vertex of S. The concept of (minimum) resolving set of a graph has proved to be useful and/or related to a variety of fields such as Chemistry [3], Robotic Navigation [2] and Combinatorial Search and Optimization [4].

This work is devoted to evaluating the so-called *metric dimension* [1, 5] of finite connected graphs, i.e., the minimum cardinality of a resolving set. Firstly we find a non-trivial universal resolving set, and then we focus our attention on cartesian products of graphs. We show some results about upper and lower bounds of the metric dimension of the product $G \times H$ of two graphs and we study in detail particular cases, such as products of complete graphs, cycles or paths. In these cases we provide exact values of the metric dimension and we also describe minimum resolving sets of cartesian product.

References.

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