Theatre A

Inversions in Combinatorics

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There are many inversion phenomena in combinatorics, such as Lagrange Inversion, inverse relations and Mobius Inversion. Implicitly, inversion phenomena also include MacMahon's master theorem, Jacobi's formula, Saalschutz's theorem and Dixon's theorem. Our philosophy is that an inversion phenomenon is an interplay of two ways to represent an object involving differentials. See [Proc. Amer. Math. Soc. 125 (1997), no. 4, 1011–1017] and [p. 255–342, Lecture Notes in Pure and Appl. Math., 226, Dekker, 2002]. We have shown that Lagrange Inversion is a phenomenon of changes of variables [Comm. Algebra 26 (1998), no. 3, 803–812]; inverse relations are simply changes of Schauder bases [J. Combin. Theory Ser. A 97 (2002), no. 2, 203–224]; Jacobi's formula is indeed a change of parameters. In the talk, we will explain also how Mobius Inversion for a locally finite partially ordered set shares the same philosophy. More precisely, the Mobius inversion formula is in fact a Lagrange inversion formula and can be interpreted as a change of Schauder bases.

Theatre B

Metric dimension for infinite graphs

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A set of vertices S resolves a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The metric dimension of a graph G is the minimum cardinality of a resolving set. In this talk we study the metric dimension of infinite graphs such that all its vertices have finite degree. We give necessary conditions for those graphs to have infinite metric dimension and characterize infinite trees with finite metric dimension. We also establish some results about the metric dimension of the cartesian product of finite and infinite graphs, and give the metric dimension of the cartesian product of several families of graphs.

Theatre C

Six-sparse Steiner triple systems

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A *Steiner triple system* of order v (STS(v)) is a pair $(V, \mathcal{B})$ where V is a non-empty set of v points and $\mathcal{B}$ is a set of 3-element subsets of v, called *blocks*, such that each pair of points occurs in precisely one block. An STS(v) exists if and only if $v \equiv 1$ or 3 (mod 6). A *configuration* is a finite set of triples where a pair of points occurs at most once. For $k \geq 4$, a Steiner triple system S is called *k-sparse* if for $4 \leq n \leq k$, every configuration in S of n blocks spans at least $n+3$ points. The terminology originates from a conjecture of Erdős: for every integer $k \geq 4$, there exists a k-sparse STS(v) for all sufficiently large admissible v. For practical purposes, k-sparseness is equivalent to the avoidance of certain n-block, $(n+2)$-point configurations for $4 \leq n \leq k$.
There is only one 4-block, 6-point configuration, namely the Pasch. The existence of 4-sparse (i.e. Pasch-free) STS(v)s for all admissible $v \geq 15$ was established in a series of papers by Brouwer (1977), Griggs, Murphy & Phelan (1990), Ling, Colbourn, Grannell & Griggs (2000) and Grannel, Griggs & Whitehead (2000). The mitre is the only Pasch-free 5-block, 7-point configuration. Systems which are 5-sparse are now known to exist for almost all admissible v and for all $v \equiv 3$ (mod 6), $v \geq 21$: Ling (1997), Fujiwara (2006), Wolfe (2005, 2008).
For 6-sparseness, the relevant 6-block, 8-point configurations are the 6-cycle and one other, which has become known as the 'crown'. Thus an STS(v) is 6-sparse iff it is Pasch-, mitre-, 6-cycle- and crown-free. We have already established the existence of infinitely many 6-sparse systems, created recursively from 29 block transitive 6-sparse STS(v)s with prime $v \equiv 7$ (mod 12): ADF, Grannell & Griggs (2007). Although we appear to be a long way from resolving the Erdős conjecture for $k = 6$, in this talk I shall explain how to create 6-sparse Steiner triple systems of order $3p$ for all sufficiently large primes $p \equiv 3$ (mod 4).