

# On the Metric Dimension of Cartesian Products of Graphs<sup>★</sup>

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## Abstract

A set  $S$  of vertices in a graph  $G$  *resolves*  $G$  if every vertex is uniquely determined by its vector of distances to the vertices in  $S$ . The *metric dimension* of  $G$  is the minimum cardinality of a resolving set of  $G$ . This paper studies the metric dimension of cartesian products  $G \square H$ . We prove that the metric dimension of  $G \square G$  is tied in a strong sense to the minimum order of a so-called doubly resolving set in  $G$ . Using bounds on the order of doubly resolving sets, we establish bounds on  $G \square H$  for many examples of  $G$  and  $H$ . One of our main results is a family of graphs  $G$  with bounded metric dimension for which the metric dimension of  $G \square G$  is unbounded.

*Key words:* graph, distance, resolving set, metric dimension, metric basis, cartesian product, Hamming graph, Mastermind, coin weighing

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## 1 Introduction

This paper undertakes a general study of the metric dimension of cartesian products of graphs. All the graphs considered are finite, undirected, simple,

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