Quasiperfect Domination in Trees [1]

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1. QP-dominating codes and the QP-chain

2. Short QP-chains

3. Trees

1. QP-dominating codes and the QP-chain

2. Short QP-chains

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▷ A set $D \subset V(G)$ of a graph G is a *dominating set* if every vertex u not in D has **at least** a neighbour in D, i.e., $\boxed{N(u) \cap D \neq \emptyset}$.
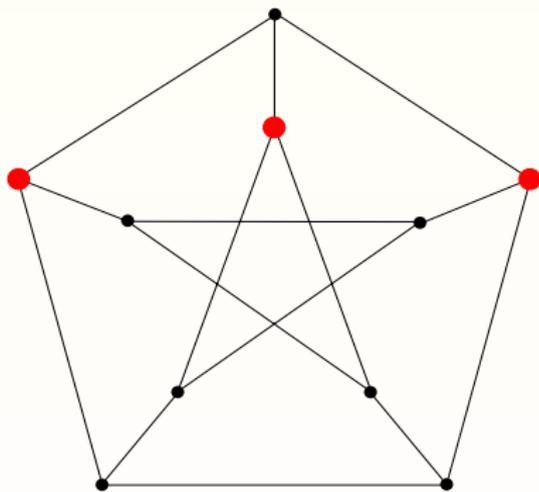
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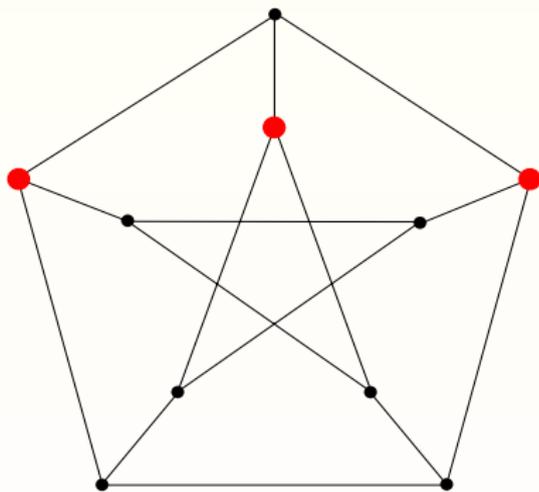▷ The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.

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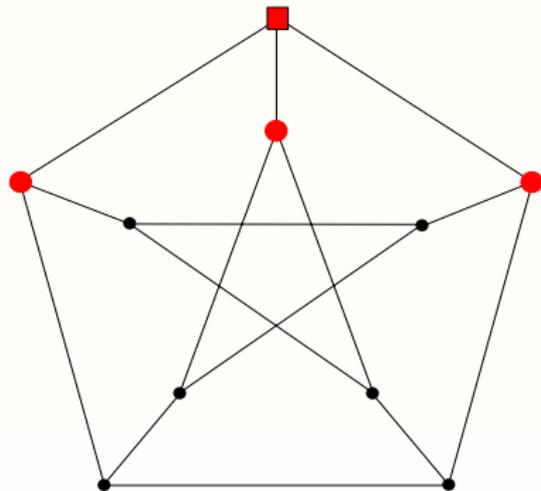• $\gamma(P) = 3$, since red vertices form a γ-code.

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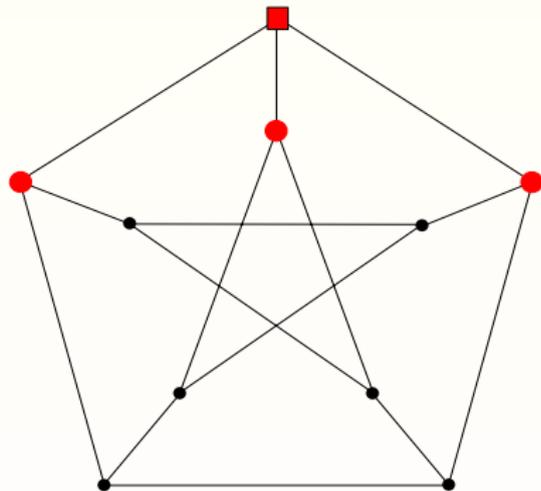
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● $\gamma_{11}(P) = 4$, since red vertices form a γ_{11}-code.
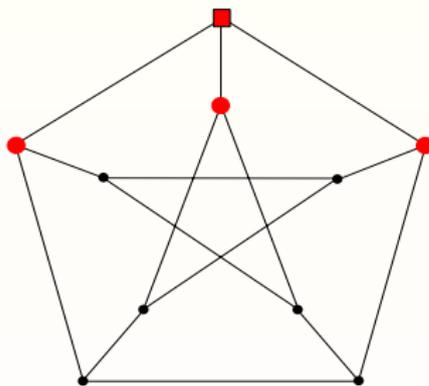
▷ A dominating set $D \subset V(G)$ of a graph G is a *k-quasiperfect dominating set* if every vertex of $V \setminus D$ has **at most** k neighbours in D, i.e., for each $u \in V \setminus D$, $\boxed{1 \leq |N(u) \cap D| \leq k}$.

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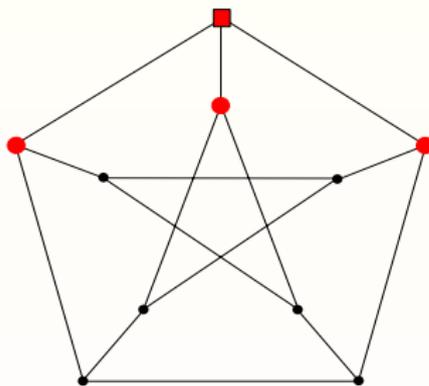
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● Note that $n = 10$, $\Delta = 3$, $\gamma_{11}(P) = \gamma_{12}(P) = 4$, $\gamma_{13}(P) = \gamma(P) = 3$.

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- A short QP-chain is called *CONSTANT* if

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1. QP-dominating codes and the QP-chain

2. ## Short QP-chains

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QP-chain of some basic graph families

G	P_n	C_n	K_n	$K_{1,n-1}$	$K_{p,n-p}$	W_n
$\Delta(G)$	2	2	$n-1$	$n-1$	$n-p$	$n-1$
$\gamma_{11}(G)$	$\lceil\frac{n}{3}\rceil$	$\lceil\frac{2n}{3}\rceil - \lfloor\frac{n}{3}\rfloor$	1	1	2	1
$\gamma_{12}(G)$	$\lceil\frac{n}{3}\rceil$	$\lceil\frac{n}{3}\rceil$	1	1	2	1
$\gamma(G)$	$\lceil\frac{n}{3}\rceil$	$\lceil\frac{n}{3}\rceil$	1	1	2	1

All of these graph families have a <u>constant</u> QP-chain,
except cycles C_{3k+2} whose QP-chain is <u>short.</u>

More graph families with a short QP-chain

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▷ $\boxed{\Delta = n - 1}$. Every graph satisfies: $\gamma_{11} = \gamma = 1$.
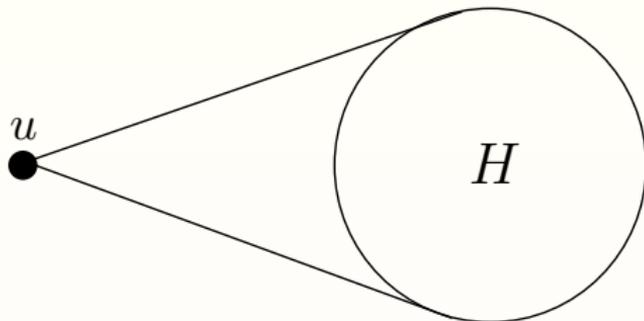
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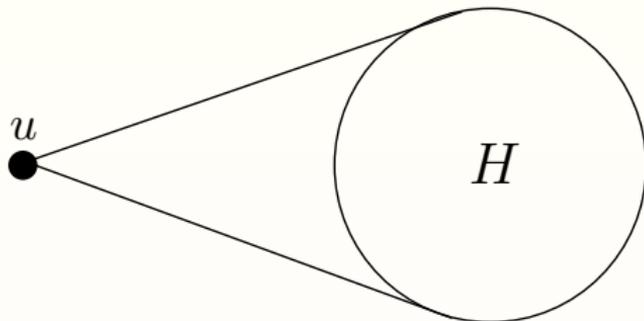
▷ $\{u\}$ is a γ_{11}-code of $G = K_1 \vee H$.

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▷ <u>Case $k = n$</u>: Take $G = P_{n-2} \vee \bar{K}_2$

▷ $\{u, w\}$ is a γ-code and the unique γ_{11}-set is the whole graph.

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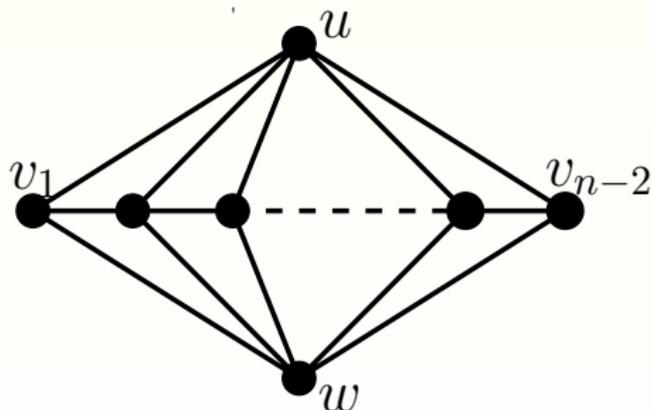
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▷ If $n \geq 7$ and $2 \leq k \leq n$, then there exists a graph G of order n and $\gamma(G) = 2$ s.t. $\Delta(G) = n-3$, $\gamma_{11}(G) = k$.

<div style="border:1px solid blue; display:inline-block;">**More graph families with a short QP-chain**</div>

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▷ If $n \geq 9$ and $3 \leq k \leq n$, then there exists a graph G of order n and $\gamma(G) = 3$ s.t. $\Delta(G) = n - 3$, $\gamma_{11}(G) = k$.

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▷ $\gamma_{11}(G) = 2 \Longleftrightarrow G$ is as in Figure.
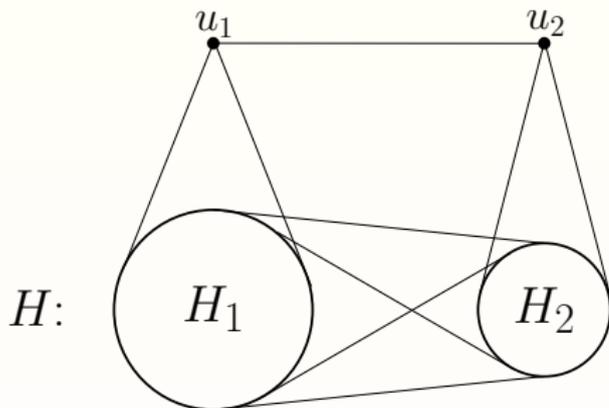
Figure: $H = H_1 \vee H_2$, $N_G(u_1) = V(H_1) + u_2$, $N_G(u_2) = V(H_2) + u_1$.

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▷ Open Problem: $3 \leq h < k$, $2n \leq 3h + k$.

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- $\gamma_{1k}(T) \leq |S_j| = |S| + j \leq \gamma(T) + \left\lceil \frac{r-k}{k} \right\rceil \leq \gamma(T) + \left\lceil \frac{\gamma(T)-k}{k} \right\rceil$

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- If $k \geq \gamma(T)$, then $\gamma_{1k}(T) = \gamma(T)$.
- If $k < \gamma(T) = a = q \cdot k + r$, with $1 \leq r \leq k$, then take this tree:

Figure: Squared vertices are a γ-code and black vertices are a γ_{1k}-code

T is a tree.

▷▷ The bound $\boxed{\gamma_{1k}(T) \leq \gamma(T) + \lceil \frac{\gamma(T)}{k} \rceil - 1}$ is tight.

Sketch of proof:

- If $k \geq \gamma(T)$, then $\gamma_{1k}(T) = \gamma(T)$.
- If $k < \gamma(T) = a = q \cdot k + r$, with $1 \leq r \leq k$, then take this tree:

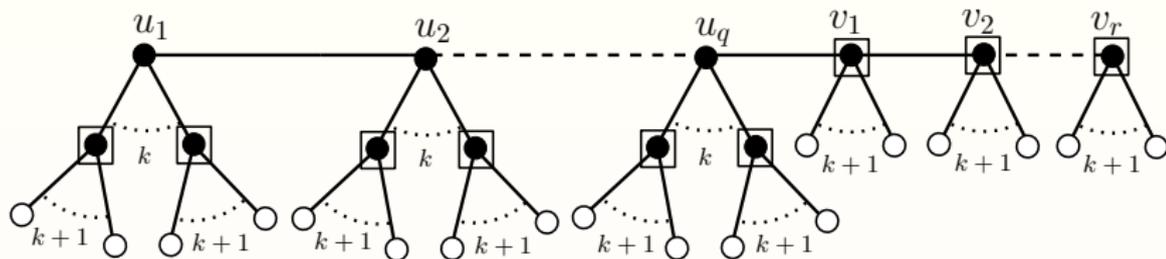Figure: Squared vertices are a γ-code and black vertices are a γ_{1k}-code

- $\gamma(T) + \left\lceil \dfrac{\gamma(T)}{k} \right\rceil - 1 = a + \left\lceil \dfrac{q \cdot k + r}{k} \right\rceil - 1 = a + q + 1 - 1 = q \cdot k + r + q = \gamma_{1k}(T)$

T is a tree: $\boxed{\gamma(T) \leq \gamma_{11}(T) \leq 2\gamma(T) - 1}$

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▷▷ A tree T satisfies $\gamma_{11}(T) = 2\gamma(T) - 1$ iff:

1. The set S of strong support vertices of T is an independent dominating set, and

2. Every vertex of any component C of $T - (S \cup L)$ has exactly one neighbor in S except one vertex that has two neighbors in S.

Two realization theorems

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Let a, b, n be integers such that $2 \leq a \leq b \leq 2a - 1$ and $n > 2b$:

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$\Delta \geq 3$:

▷ There exists a tree with maximum degree Δ satisfying each one of the $2^{\Delta-1}$ possible combinations of the QP-chain.

Two tree families with a short QP-chain

$\star$ A *caterpillar* is a tree s.t. the removal of all its leaves gives rise to a path.
$\star$ A *k-ary tree* is a rooted tree such that each vertex has at most k children.
$\star$ A *full k-ary tree* is a k-ary tree such that all vertices that are not leaves have exactly k children.

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$\triangleright$ If T is a caterpillar, then:

$$2\gamma(T) > \gamma_{11}(T) \geq \gamma_{12}(T) = \gamma(T)$$

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$\triangleright$ If T is full k-ary tree, then:

$$n - \ell(T) = \gamma_{11}(T) = \cdots = \gamma_{1,k-1}(T) > \gamma_{1,k}(T) = \gamma_{1,k+1}(T) = \gamma(T)$$

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GRACIAS/OBRIGADO/THANKS

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