Generalizing the Krein-Milman property in graph convexity spaces: a short survey

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Abstract

A *convexity* on a non-empty finite set V is a family $\mathcal{C}$ of subsets of V (to be regarded as convex sets), which is closed under intersections and which contains both V and the empty set. The pair $(V, \mathcal{C})$ is called a *convexity space*. The smallest convex set containing a set $A \subseteq V$ is said to be the *convex hull* of A. Given a convex set $W \subseteq V$, a vertex $v \in W$ is called an *extreme vertex* of W if the set $W \setminus \{v\}$ is also convex. A *convex geometry* is a convexity space satisfying the so-called *Krein-Milman* property: Every convex set of V is the convex hull of its extreme vertices. A *Krein-Milman type property* is any property similar to the previous one in which either the set of vertices (the extreme vertex set) or the operator (the convex hull one), or both, are replaced. In this paper, we survey the state-of-art of this type of properties for graph convexity spaces.

Key words: Boundary; Contour; Extreme vertex; Geodetic set; Graph convexity space; Hull set; Krein-Milman property; Monophonic set; Steiner set.

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