

# Algebraic properties of a digraph and its line digraph

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## Abstract

Let  $G$  be a digraph,  $LG$  its line digraph and  $A(G)$  and  $A(LG)$  their adjacency matrices. We present relations between the Jordan Normal Form of these two matrices. Besides, we study the spectra of those matrices and obtain a relationship between their characteristic polynomials that allows to relate properties of  $G$  and  $LG$  regarding the number of cycles of a given length.

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