On the Metric Dimension of Infinite Graphs[☆]

J. Cáceres[*,a], C. Hernando[b], M. Mora[b], I.M. Pelayo[b], M.L. Puertas[a]

[a] *Universidad de Almería, Almería, Spain*
[b] *Universitat Politècnica de Catalunya, Barcelona, Spain*

Abstract

A set of vertices S *resolves* a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The *metric dimension* of a graph G is the minimum cardinality of a resolving set. In this paper we study the metric dimension of infinite graphs such that all its vertices have finite degree. We give necessary conditions for those graphs to have finite metric dimension and characterize infinite trees with finite metric dimension. We also establish some results about the metric dimension of the cartesian product of finite and infinite graphs, and give the metric dimension of the cartesian product of several families of graphs.

Key words: infinite graph, locally finite graph, resolving set, metric dimension, cartesian product

1. Introduction

Throughout this paper a *graph* G is an ordered pair of disjoint sets (V, E) where V is nonempty and E is a subset of unordered pairs of V. The *vertices* and *edges* of G are the elements of $V = V(G)$ and $E = E(G)$ respectively.

We say that a graph G is *finite* (resp. *infinite*) if the set $V(G)$ is finite (resp. infinite). The degree of a vertex $u \in V(G)$ is the number of edges containing u and is denoted by $\deg_G(u)$, or simply $\deg(u)$ if the graph G

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*Corresponding author

Email addresses: `jcaceres@ual.es` (J. Cáceres), `carmen.hernando@upc.edu` (C. Hernando), `merce.mora@upc.edu` (M. Mora), `ignacio.m.pelayo@upc.edu` (I.M. Pelayo), `mpuertas@ual.es` (M.L. Puertas)