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Geodeticity of the contour of chordal graphs[☆]

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Abstract

A vertex v is a *boundary vertex* of a connected graph G if there exists a vertex u such that no neighbor of v is further away from u than v. Moreover, if no vertex in the whole graph $V(G)$ is further away from u than v, then v is called an *eccentric vertex* of G. A vertex v belongs to the *contour* of G if no neighbor of v has an eccentricity greater than the eccentricity of v. Furthermore, if no vertex in the whole graph $V(G)$ has an eccentricity greater than the eccentricity of v, then v is called a *peripheral vertex* of G. This paper is devoted to study these kinds of vertices for the family of chordal graphs. Our main contributions are, firstly, obtaining a realization theorem involving the cardinalities of the periphery, the contour, the eccentric subgraph and the boundary, and secondly, proving both that the contour of every chordal graph is geodetic and that this statement is not true for every perfect graph.

Keywords: Boundary; Contour; Convex hull; Convexity; Geodetic set; Chordal graph; Perfect graph

1. Introduction

The extraordinary development during the last few decades of a number of discrete and combinatorial mathematical structures has led to the creation and study of distinct analogies and generalizations of a number of classical concepts, ideas, and methods from continuous mathematics. Among them, the notion of convex set of a metric space and the convex hull operator play a significant role [12]. Since connected graphs can be seen as metric spaces just by considering their shortest paths, this fact has led to the study of the behavior of these structures as convexity spaces [5,12,14].

All graphs in this paper are finite, undirected, simple and connected. For undefined basic concepts we refer the reader to introductory graph theoretical literature, e.g., [13]. Given vertices u, v in a graph $G=(V,E)$, the *geodetic interval* $I[u,v]$ is the set of vertices of all u–v shortest paths. Given $W \subseteq V$, the *geodetic closure* $I[W]$ of W is the union

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