Superconnected Digraphs and Graphs with Small Conditional Diameters

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Abstract

The conditional diameter D_ν of a digraph G measures how far apart a pair of vertex sets V_1 and V_2 can be in such a way that the minimum out-degree and the minimum in-degree of the subdigraphs induced by V_1 and V_2, respectively, are at least ν. Thus, D_0 is the standard diameter and $D_0 \geq D_1 \geq \ldots \geq D_\delta$, where δ is the minimum degree. We prove that if $D_\nu \leq 2\ell - 3$, where ℓ is a parameter related to the shortest paths, then G is maximally connected, superconnected or has a good superconnectivity, depending only on whether ν is equal to $\lceil \delta/2 \rceil$, $\lceil (\delta-1)/2 \rceil$, $\lceil (\delta-1)/3 \rceil$, respectively. In the edge case, it is enough that $D_\nu \leq 2\ell - 2$. The results for graphs are obtained as a corollary of those for digraphs, because in the undirected case, $\ell = \lfloor (g-1)/2 \rfloor$, g being the girth.

Key words. digraphs, connectivity, superconnectivity, fault-tolerance, diameter, girth.

AMS subject classification. 05C40, 05C20

References

[1] M. Aigner, On the linegraph of a directed graph, Math Z 102 (1967), 56-61.

[2] C. Balbuena and A. Carmona, On the connectivity and superconnectivity of Bipartite digraphs and graphs, Ars Combin 61 (2001), 3–21.

[3] C. Balbuena, A. Carmona, J. Fàbrega, and M.A. Fiol, On the connectivity and the conditional diameter of graphs and digraphs, Networks 28 (1996), 97–105.

[4] F.T Boesch, Synthesis of reliable networks—A survey, IEEE Trans Reliab 35 (1986), 240–246.

[5] F. Boesch and R. Tindell, Circulants and their connectivities, J Graph Theory 8 (1984), 487–499.

[6] G. Chartrand and L. Lesniak, Graphs and Digraphs, 3rd ed., Chapman and Hall, 1996.

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[7] J. Fàbrega and M. A. Fiol, Maximally connected digraphs, J Graph Theory 13 (1989), 657–668.

[8] J. Fàbrega and M. A. Fiol, Bipartite graphs and digraphs with maximum connectivity, Discr Appl Math 69 (1996), 269–278.

[9] M.A. Fiol, J.L.A. Yebra and I. Alegre, Line digraph iterations and the (d, k) digraph problem, IEEE Trans Comput C-33 (1984), 400–403.

[10] M.A. Fiol, J. Fàbrega, and M. Escudero, Short paths and connectivity in graphs and digraphs, Ars Combin 29B (1990), 17–31.

[11] D. Geller and F. Harary, Connectivity in digraphs, Lecture Notes in Mathematics 186, Springer, Berlin (1970), 105–114.

[12] F. Harary, Conditional connectivity, Networks 13 (1983), 347–357.

[13] S.M. Reddy, J.G. Kuhl, S.H. Hosseini, and H. Lee, On digraphs with minimum diameter and maximum connectivity, Proc of the 20th Ann Allerton Conf (1982), 1018–1026.

[14] T. Soneoka, Super edge-connectivity of dense digraphs and graphs, Discr Appl Math 37/38 (1992), 511–523.

[15] T. Soneoka, H. Nakada, and M. Imase, Sufficient conditions for dense graphs to be maximally connected, Proc ISCAS85, IEEE Press, New York (1985), 811–814.

[16] T. Soneoka, H. Nakada, M. Imase, and C. Peyrat, Sufficient conditions for maximally connected dense graphs, Discr Math 63 (1987), 53–66.