On the connectivity of generalized p-cycles

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Abstract

A generalized p-cycle is a digraph whose set of vertices is partitioned in p parts that are cyclically ordered in such a way that the vertices in one part are adjacent only to vertices in the next part. In this work, we mainly show the two following types of conditions in order to find generalized p-cycles with maximum connectivity:

1. For a new given parameter ℓ, related to the number of short paths in G, the diameter is small enough.
2. Given the diameter and the maximum degree, the number of vertices is large enough.

For the first problem it is shown that if $D \leq 2\ell + p - 2$, then the connectivity is maximum. Similarly, if $D \leq 2\ell + p - 1$, then the edge-connectivity is also maximum. For problem two an appropriate lower bound on the order, in terms of the maximum and minimum degree, the parameter ℓ and the diameter is deduced to guarantee maximum connectivity.

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