September 20 17h50m Room B

Neighbor-Locating Colorings in Graphs

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VERTEX PARTITION

$G = (V, E)$ connected graph

$\Pi = \{S_1, \ldots, S_k\}$ **vertex partition**: $\begin{cases} S_i \neq \emptyset \\ \bigcup_{i=1}^{k} S_i = V \end{cases}$

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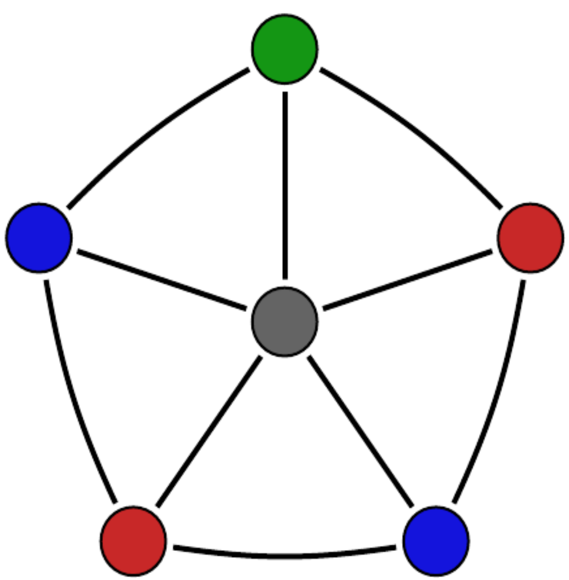
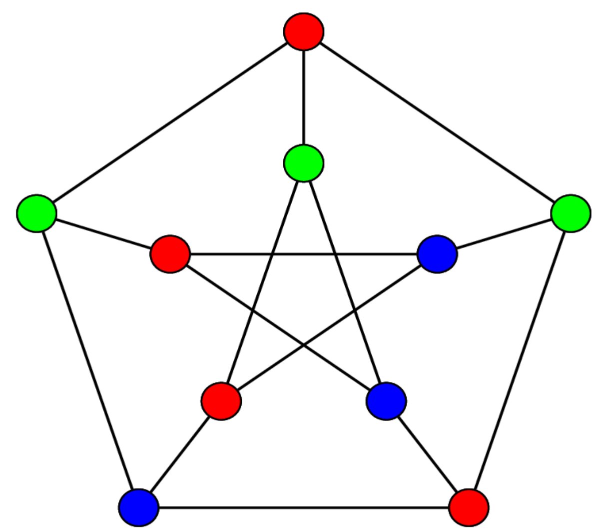
- Stable k-partition $=$ k-coloring



$\chi(W_5) = 4$



$\chi(\mathcal{P}) = 3$

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- S minimum dominating $\rightarrow \{S, V - S\}$ dominating.

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- u, v twins: $u \in S_i \Rightarrow v \notin S_i$

PARAMETERS (general case)

- $\beta_p(G)$: $\begin{cases} \star\,\text{minimum ML partitions} \\ \star\,\text{Chartrand et al. , 2000} \\ \star\,\text{partition dimension} \end{cases}$

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[$\beta_p = \eta_p$ iff G has a minimum ML-partition without interior vertices]

- $\lambda_p(G)$: $\begin{cases} \star\,\text{minimum NL partitions} \\ \star\,\text{partition NL number} \\ \star\,\beta_p \leq \eta_p \leq \lambda_p \\ \star\,diam(G) = 2 \Rightarrow \eta_p = \lambda_p \end{cases}$

PARAMETERS (stable case)

- $\chi_{ML}(G)$: $\begin{cases} \star\,\text{minimum ML colorings} \\ \star\,\text{Chartrand, Henning, Slater et al., 2002} \\ \star\,\text{ML} - \text{chromatic number} \\ \star\,\{\eta_p, \chi\} \leq \chi_{ML} \end{cases}$

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- $\chi_{NL}(G)$: $\begin{cases} \star\,\text{minimum NL colorings} \\ \star\,\text{NL} - \text{chromatic number} \\ \star\,\eta_p \leq \{\lambda_p, \chi_{ML}\} \leq \chi_{NL} \\ \star\,\omega \leq \chi \leq \chi_{ML} \leq \chi_{NL} \\ \star\,diam(G) = 2 \Rightarrow \chi_{ML} = \chi_{NL} \end{cases}$

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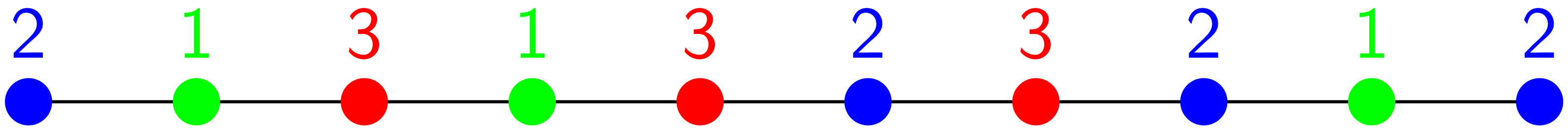

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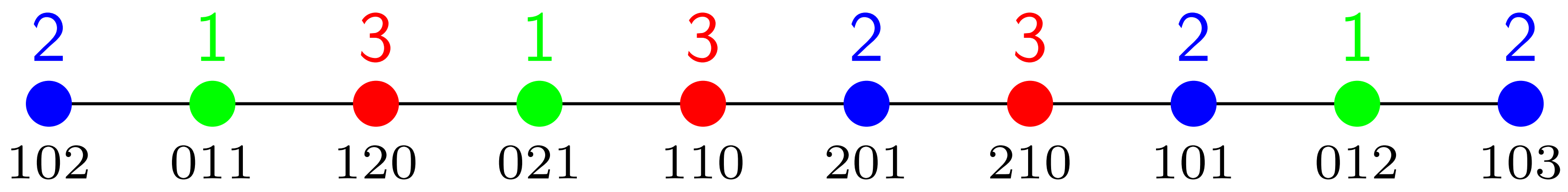

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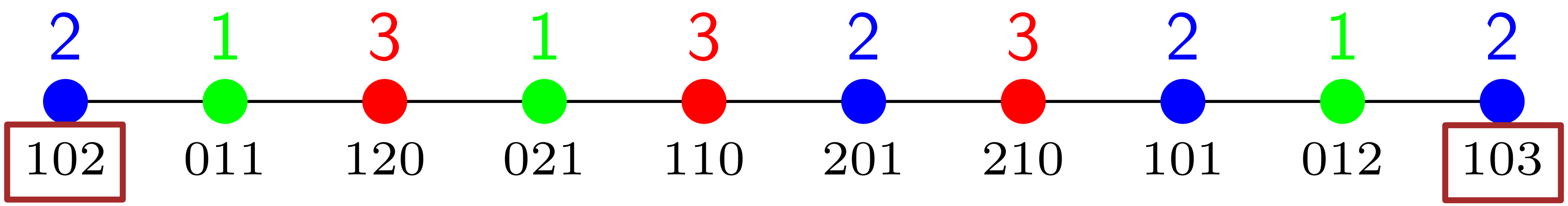

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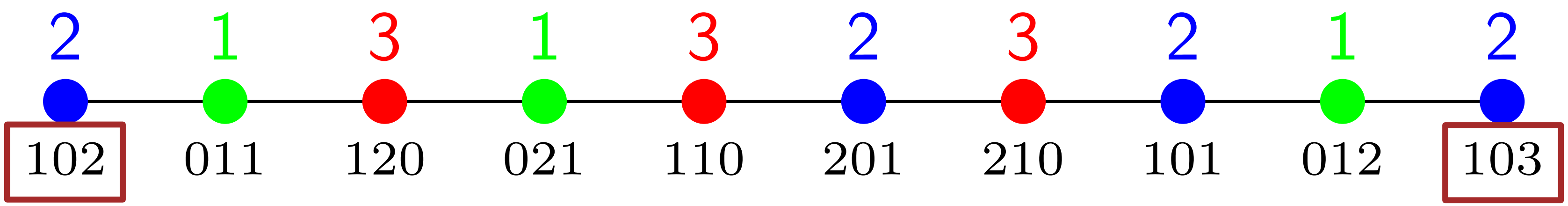

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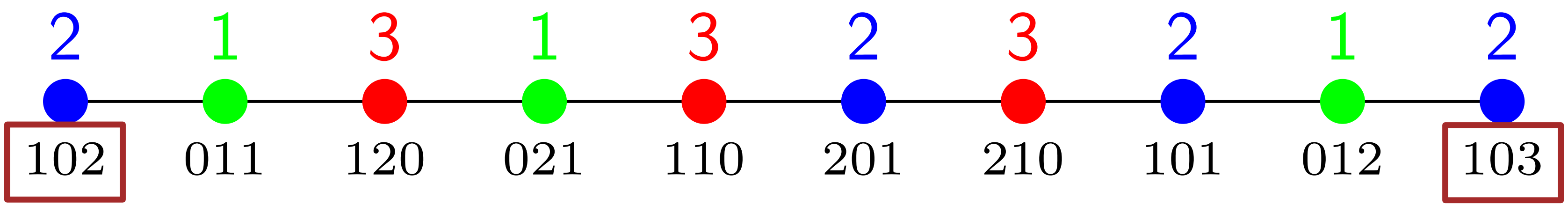

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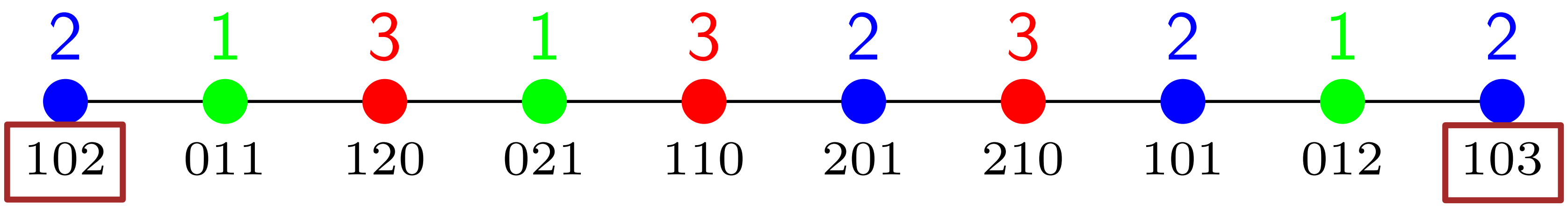

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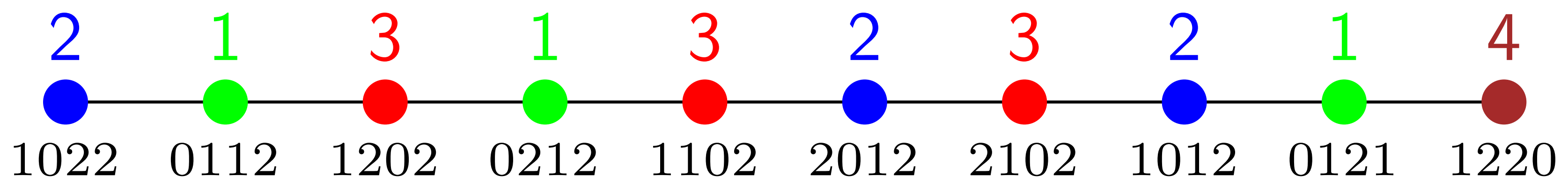

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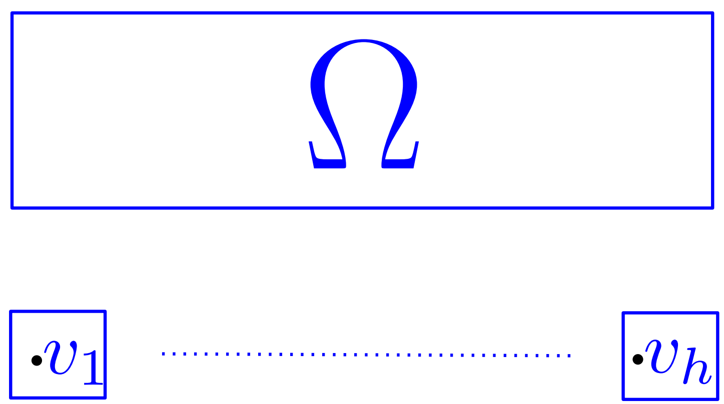

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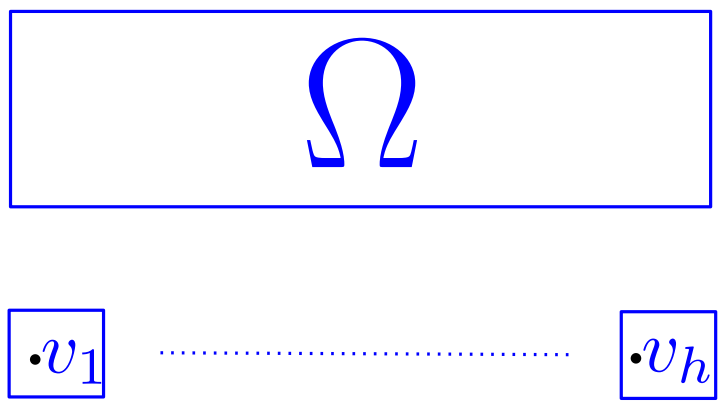

$\Pi = \{\{v_1\}, \ldots, \{v_h\}, \Omega\}$ is an NL coloring

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- $T \neq P_4 \Rightarrow \chi_{NL}(T) \neq n - 1$

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- Unicyclic: $n \leq \frac{1}{2}(k^3 + k^2 - 2k)$

$\chi_{NL} = k \Rightarrow n \leq k \cdot (2^{k-1} - 1)$ [tightness]

For every $k \geq 3$, let $G_k = (V, E)$ is the graph such that:

- $V = \bigcup_{i=1}^{k} V_i$, where

  (1) $V_i \subset \{0, 1, 2\}^k$, for every $i \in \{1, \ldots, k\}$.
  (2) $x = x_1 \ldots x_k \in V_i$ if and only if $x_i = 0$ and, for some $j \in \{1, \ldots, k\}$, $x_j = 1$.

- $xy \in E$ if and only if $i \neq j$, $x_j = 1$ and $y_i = 1$.

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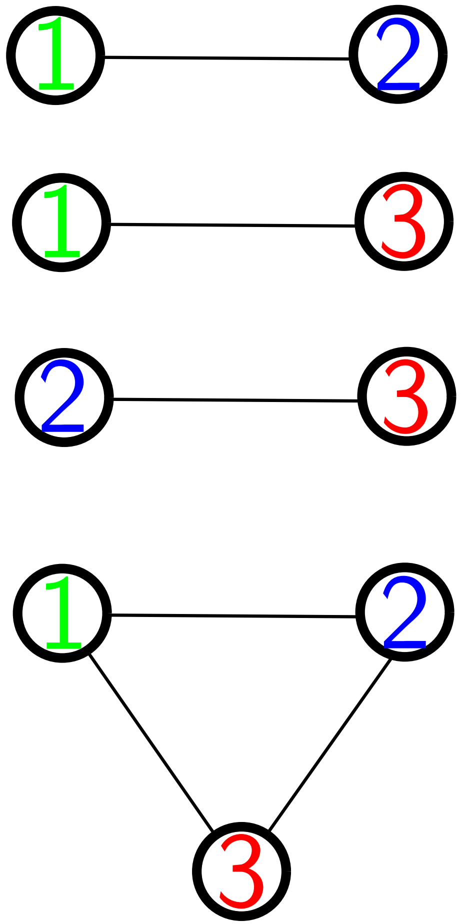
- Let G be s.t. $n = k \cdot (2^{k-1} - 1)$ and $\chi_{NL}(G) = k$. Then, G is a subgraph of G_k and $k \cdot (k-1) \cdot 2^{k-3} = m_k \leq m \leq M_k = k \cdot (k-1) \cdot 2^{2k-5}$

$\chi_{NL} = 3 \Rightarrow n \leq 3 \cdot (2^{3-1} - 1) = 9$ [tightness]



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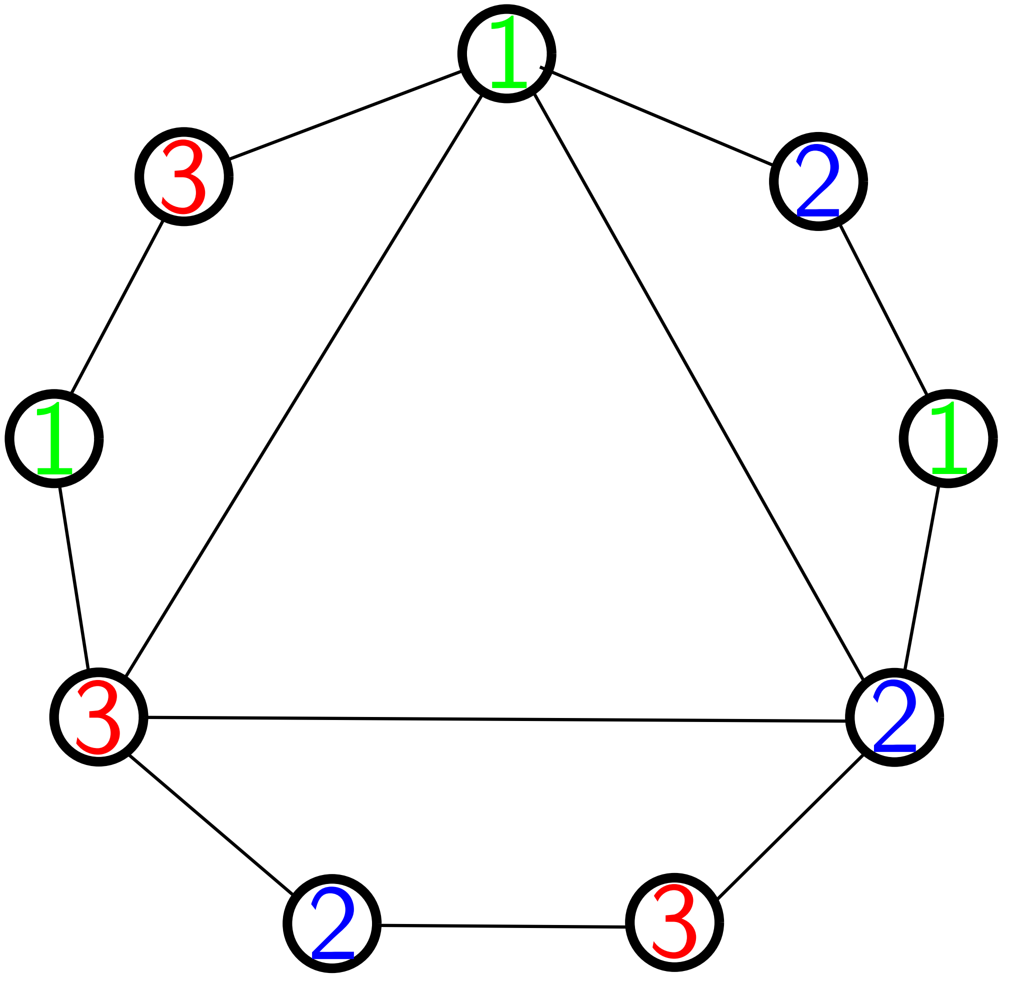
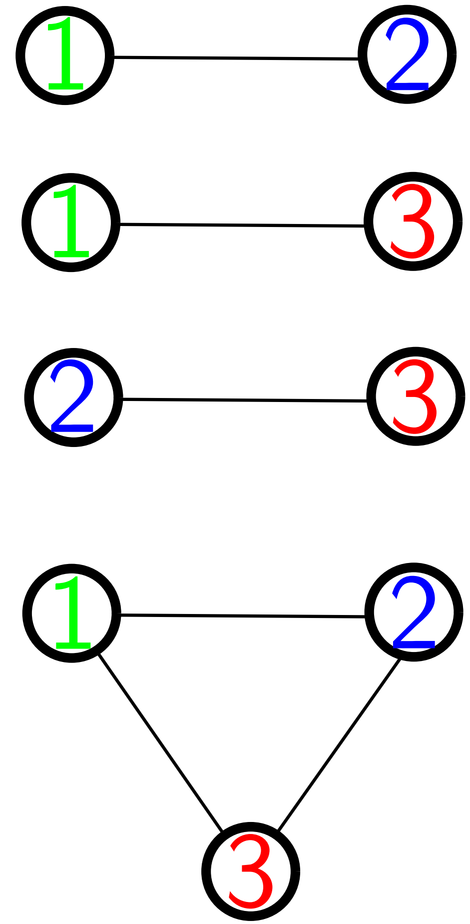


$n = 9,\ m = 12,\ c = 0$

G_3



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$n \geq 10$, $\rho(x) = x\binom{x}{2}$:

- $\rho(k-1) < n \leq \rho(k)$:

$$\chi_{NL}(P_n) = \chi_{NL}(C_n) = k$$

NL-COLORINGS: TREES

T, tree of order n and $\chi_{NL}(T) = k$

- $n = n_1 + n_2 + n_3$, where T has:

$$\begin{cases} n_1 \text{ leaves,} \\ n_2 \text{ vertices of degree 2,} \\ n_3 \text{ vertices of degree at least 3.} \end{cases}$$

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- $n_3 \leq n_1 - 2 \leq k^2 - k - 2$

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- $n_2 \leq k\binom{k-1}{2} = \frac{k(k-1)(k-2)}{2} = \frac{k^3-3k^2+2k}{2}$
- $n_3 \leq n_1 - 2 \leq k^2 - k - 2$

- $n \leq \frac{1}{2}(k^3 + k^2 - 2k - 4)$ [tight, for $k \geq 6$]

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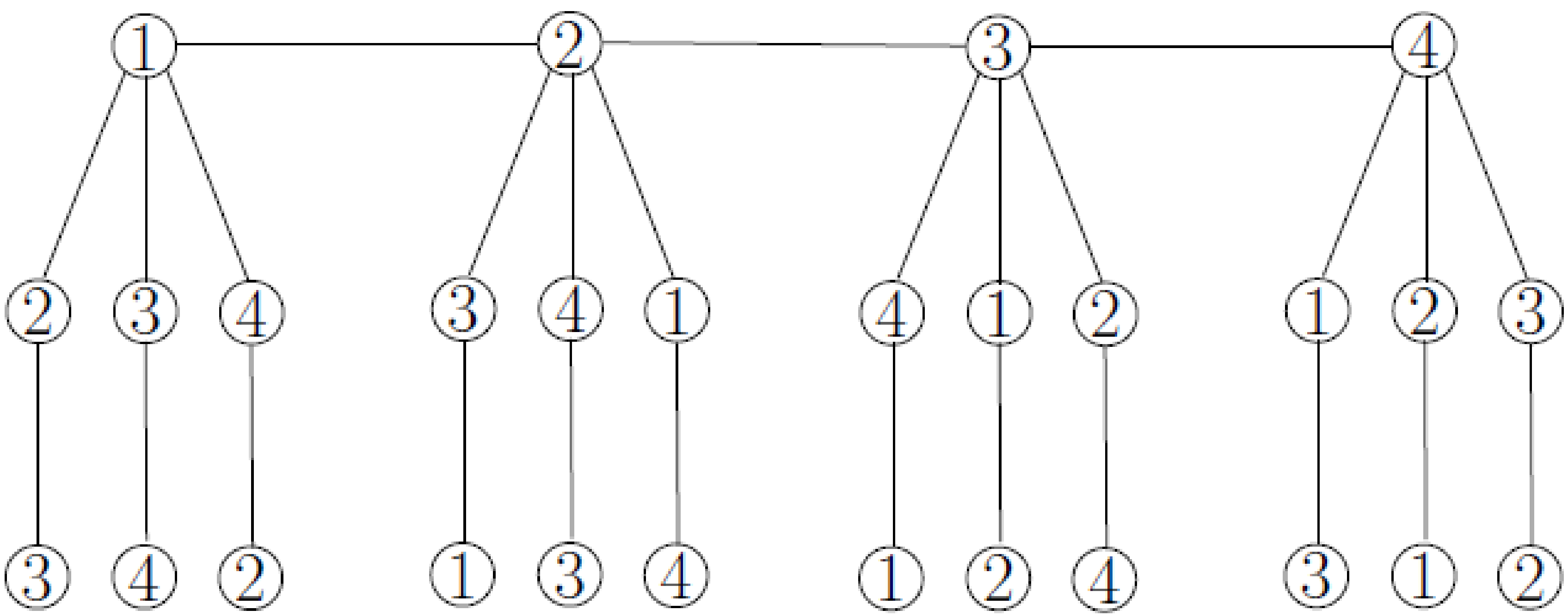

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- $\chi_{NL} = \begin{cases} \rho(G), \text{ if } \Lambda = \emptyset \\ \max\{\rho(G), |U| + 1\}, \text{ if } \Lambda \neq \emptyset \end{cases}$

Thank you for your attention!

CID 2017

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