Neighbor-Locating Colorings in Pseudotrees [1]

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▷ $G = (V, E)$ is a connected graph.

▷ A <u>dominating</u> set $S \subset V$ is a *neighbor-locating-dominating set* [2] if for every two vertices $u, v \in V \setminus S$ and some vertex $w \in S$,

$$w \in N(u) \text{ and } w \notin N(v)$$

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[3] An NLD-set of minimum cardinality.

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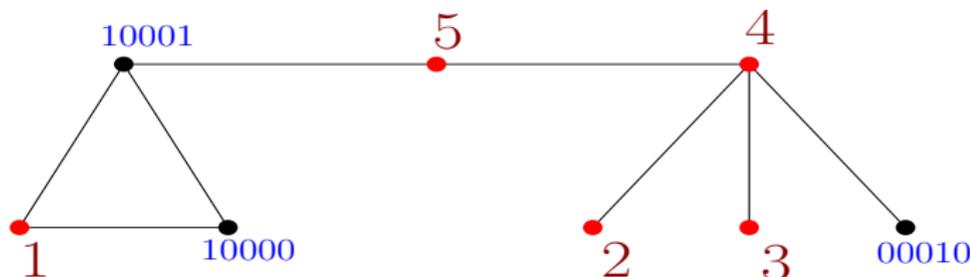▷ The *NLD number* $\lambda(G)$ is the cardinality of a *λ-code*[3].

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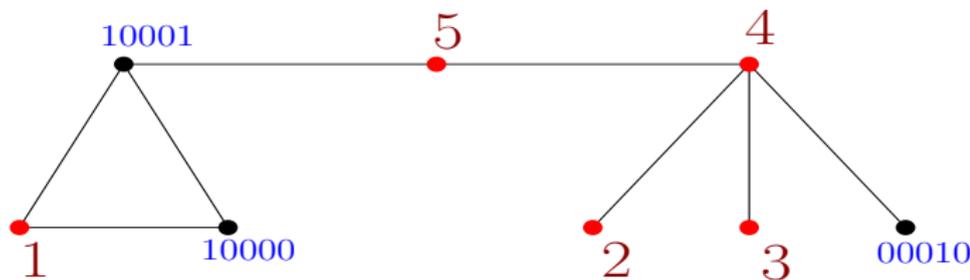

● $\boxed{\lambda(G) = 5}$ ⇒: $S = \{1, 2, 3, 4, 5\}$ is a λ-code.

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⇒ If $\lambda(G) = \lambda$ and $S = \{u_1, \ldots, u_\lambda\}$ is a λ-code of G, then $\{\{u_1\}, \ldots, \{u_\lambda\}, V \setminus S\}$ is an NLD-partition of G.

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is defined as follows:

$$x_i = \begin{cases} 0, & \text{if } x \in S_i; \\ 1, & \text{if } x \in N(S_i) \setminus S_i; \\ 2, & \text{if } x \notin N[S_i]. \end{cases}$$

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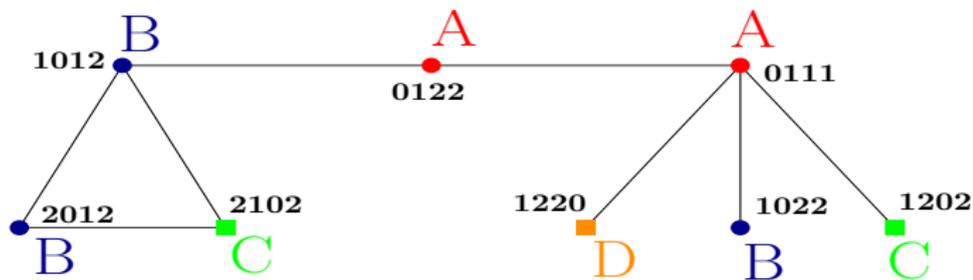
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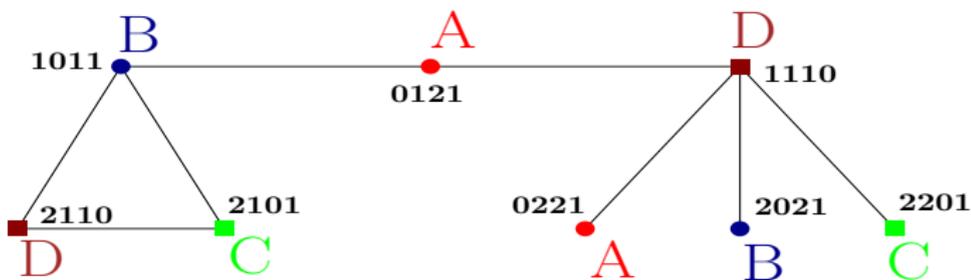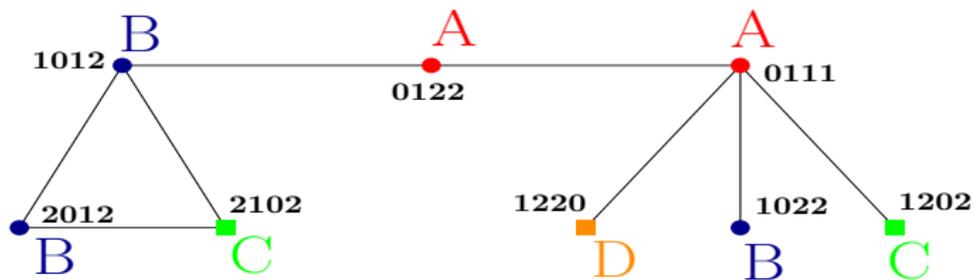- Π is an *NLD-partition* iff it is dominating and for every $u, v \in V$,
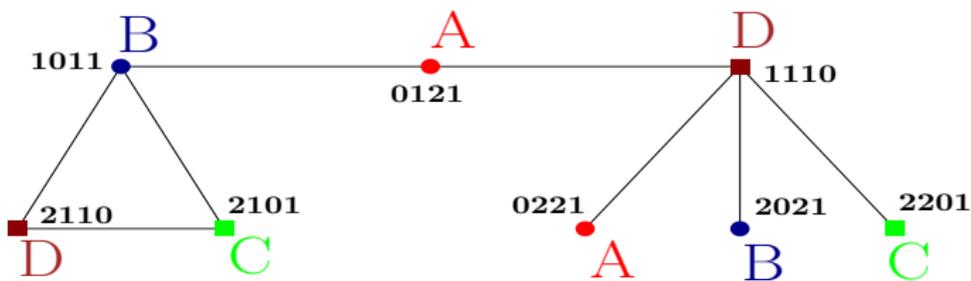
$$\boxed{nr(u|\Pi) \neq nr(v|\Pi)}$$

- $\lambda(G) = 5$ $\lambda_p(G) = 4$ $4 \leq \lambda_p(G) \leq \lambda(G) + 1 = 6$

▷ The second NLD-partition is a ***proper coloring***.

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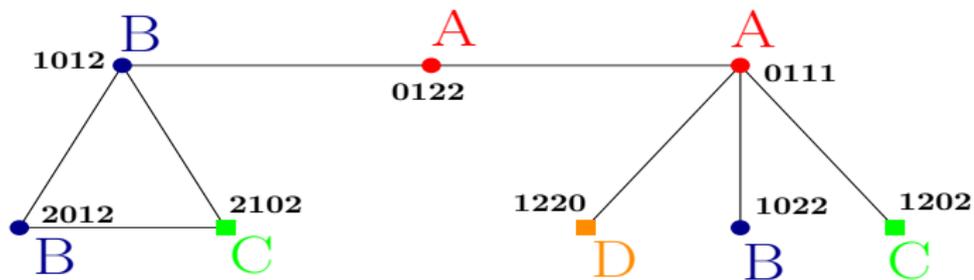
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[2] There are $2^{k-1} - 1$ k-tuples with the i-th component equal to 0 and the remaining components equal to 1 or 2, but not all them equal to 2.

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⇒ [1] For every $x \in S_i$, the i-th component of the k-tuple $nr(x|\Pi)$ is 0,

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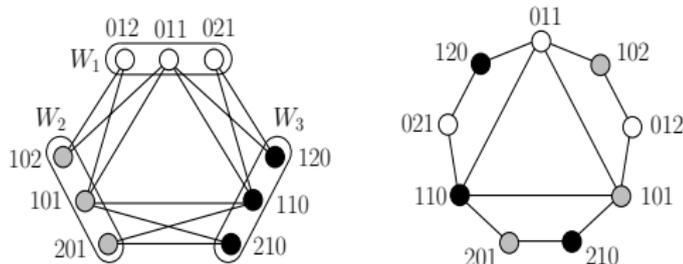
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  - V_k is the set of all k-tuples in the alphabet $\{0, 1, 2\}$ having exactly one 0 and at least one 1.
  - For every $i \in [k]\}$, W_i is the set of k-tuples $(x_1 \ldots x_k) \in V_k$ such that $x_i = 0$, so that $\{W_1, \ldots, W_k\}$ is a coloring of V_k:
  - For every $x, y \in V_k$, if $x = x_1 \ldots x_k \in W_i$ and $y = y_1 \ldots y_k \in W_j$, then $xy \in E_k$ if and only if $i \neq j$, $x_j = 1$ and $y_i = 1$.

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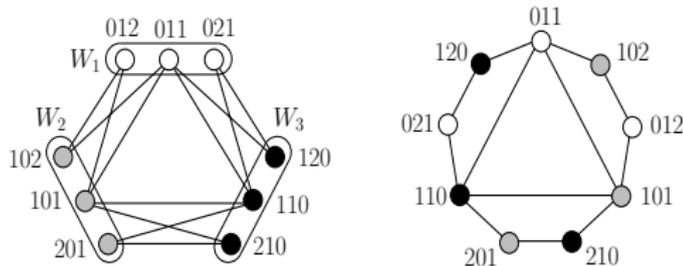
$\Rightarrow$ G_k is a connected graph of order $n_k = k \cdot [2^{k-1} - 1]$ and $\chi_{NL}(G_k) = k$.

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★ $\mathcal{G}$ is the set of all graphs $G = G^* \vee 2K_2$, the join of $2K_2$ and a complete multipartite graph G^* of order $n - 4$.

★ $\mathcal{F}$ is the set of all graphs G such that, for some vertex $v \in G$, $G - v$ is a complete multipartite graph satisfying either

  ▸ $a_i \in \{0, n_i\}$ for every $i \in [h]$, and $|\{i \in [h]\} \mid a_i = 0\}| \geq 2$, or
  ▸ there is exactly one integer $i \in [h]$ such that $a_i \notin \{0, n_i\}$, and $a_i = n_i - 1$.

  where $G(V) = V_1 \cup \ldots \cup V_h$, $n_i = |V_i|$ and $a_i = |N(v) \cap V_i|$.

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⇒ Same proof, but in this case: $n_1 \geq n_{\geq 3} + 2$.

- The bound $n \leq \boxed{\dfrac{k^3 + k^2 - 2k}{2}}$ is tight for unicyclic graphs and $k \geq 5$.
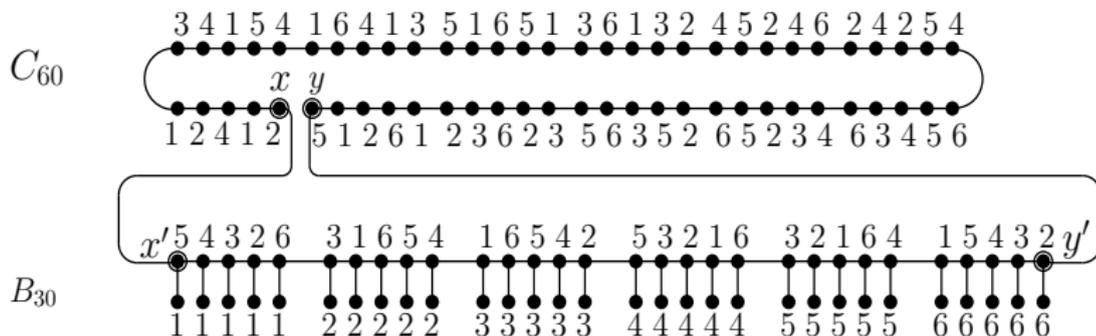
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$\Rightarrow$ $a(k) = k\binom{k-1}{1}$, $b(k) = k\binom{k-1}{2}$:

[1] The comb $B_{a(x)}$ has order $2a(x)$ and $\chi_{NL}(B_{a(x)}) = k$.

[2] The cycle $C_{b(x)}$ has order $b(x)$ and $\chi_{NL}(C_{b(x)}) = k$.

[3] $U(k)$ is the unicyclic graph of order $2a(x) + b(x) = \frac{k^3 + k^2 - 2k}{2}$ obtained from $B_{a(x)}$ and $C_{b(x)}$ as shown in the Figure (for $k = 6$).
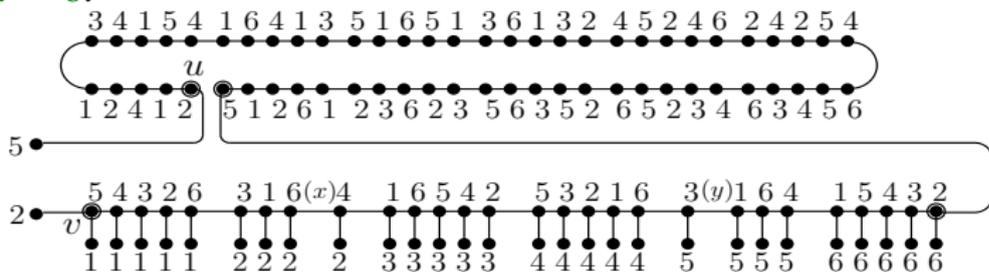

[4] There is a k-NL-coloring of $B_{a(x)}$ and a k-NL-coloring of $C_{b(x)}$, such that the corresponding k-coloring is also a k-NL-coloring of $U(k)$.

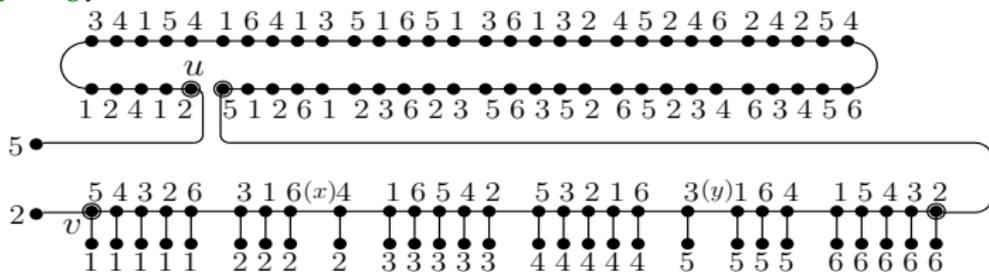- The bound $n \leq \boxed{\dfrac{k^3 + k^2 - 2k - 4}{2}}$ is tight for trees and $\boxed{k \geq 6}$.

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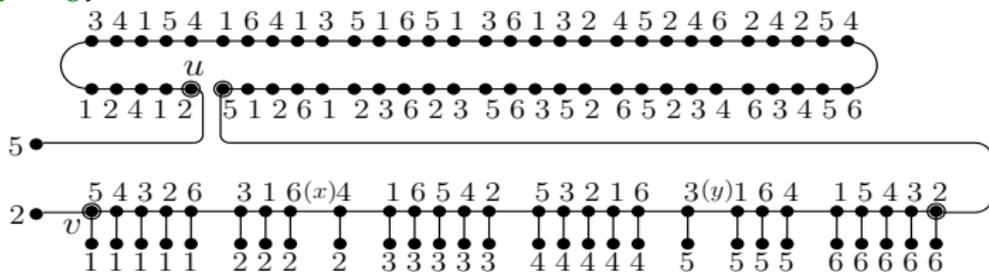
$\Rightarrow$ Case $k = 6$:

$\triangleright$ A tree T_1 of order 28 and $\chi_{NL}(T_1) = 4$ (the general bound is 28):
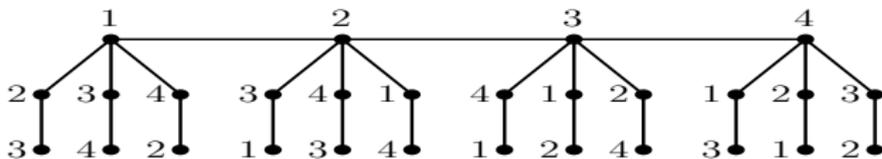
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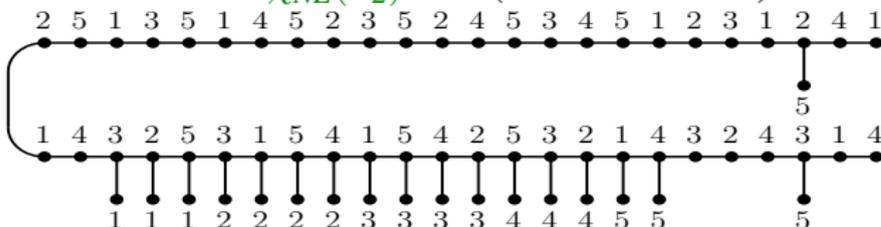$\triangleright$ A tree T_1 of order 28 and $\chi_{NL}(T_1) = 4$ (the general bound is 28):
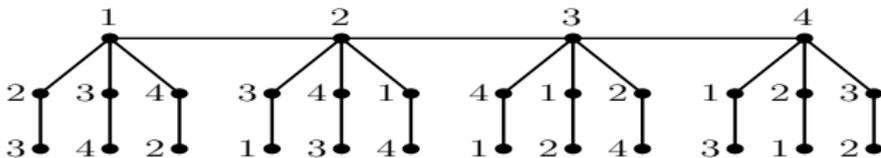
$\triangleright$ A tree T_2 of order 66 and $\chi_{NL}(T_2) = 5$ (the bound is 68):

[6]every vertex of color-degree 1 has a neighbor of color-degree 1.

Ignacio M. Pelayo (U.P.C.)   *Neighbor-Locating Colorings in Pseudotrees*   3 September 2018   13 / 14

- If $\boxed{\Delta = 2}$ and $\chi_{NL}(G) = k$, then

$$n \leq \ell(k) = k\left[\binom{k-1}{1} + \binom{k-1}{2}\right] = \frac{k^3 - k^2}{2}.$$

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$\Rightarrow$ [1] If $k \ge 3$, then $\chi_{_{NL}}(C_{\ell(k)-1}) = k+1$.

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  [3] For every $k \ge 3$, there is a 1-paired[6] k-NL-coloring of $C_{\ell(k)}$.

  [4] If there is a 1-paired $(k-1)$-NL-coloring of $C_{\ell(k-1)}$, then there is a 1-paired k-NL-coloring of C_n whenever $\ell(k-1) < n \le \ell(k) - 2$.

  [5] If there is a 1-paired k-NL-coloring of C_n, then there is a k-NL-coloring of P_n.

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$\Gamma(5) = 68$.

There is a tree T of order 66 and $\chi_{NL}(T) = 5$.

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THANKS