

Every cage with girth at least 10 is 4-connected

X. Marcote*, C. Balbuena, I. Pelayo, J. Fàbrega
Universitat Politècnica de Catalunya

Abstract

A (δ, g) -cage is a regular graph of degree δ and girth g with the least possible number of vertices. It was proved by Fu, Huang and Rodger that every $(3, g)$ -cage is 3-connected. Moreover, the same authors conjectured that all (δ, g) -cages are δ -connected for every $\delta \geq 3$. As a first step towards the proof of this conjecture, Jiang and Mubayi showed that every (δ, g) -cage with $\delta \geq 3$ is 3-connected. In this paper, we prove that every (δ, g) -cage with $\delta \geq 4$ and girth $g \geq 10$ is 4-connected, which can be seen as a further step towards the proof of the aforementioned *conjecture*.

Key words. cage, connectivity, cutset.

MSC 2000. 05C35, 05C40.

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* francisco.javier.marcote@upc.es m.camino.balbuena@upc.es ignacio.m.pelayo@upc.es matjfc@mat.upc.es