

Every Cubic Cage is quasi 4-connected

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Abstract

A (δ, g) -cage is a regular graph of degree δ and girth g with the least possible number of vertices. It was proved by Fu, Huang and Rodger that every $(3, g)$ -cage is 3-connected. Moreover, the same authors conjectured that all (δ, g) -cages are δ -connected for every $\delta \geq 3$. As a first step towards the proof of this conjecture, Jiang and Mubayi showed that every (δ, g) -cage with $\delta \geq 3$ is 3-connected. A 3-connected graph G is called *quasi 4-connected* if for each cutset $S \subset V(G)$ with $|S| = 3$, S is the neighbourhood of a vertex of degree 3 and $G - S$ has precisely two components. In this paper we prove that every $(3, g)$ -cage with $g \geq 5$ is quasi 4-connected, which can be seen as a further step towards the proof of the aforementioned *conjecture*.

Key words. quasi 4-connected graph, cage, connectivity, superconnectivity, cutset.

MSC 2000. 05C35, 05C40.

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