

On geodetic sets formed by boundary vertices

José Cáceres^a Carmen Hernando^b Mercè Mora^d
Ignacio M. Pelayo^c María L. Puertas^a Carlos Seara^{d,2}

^a*Departamento de Estadística y Matemática Aplicada, Universidad de Almería, Almería, Spain, jcaceres@ual.es, mpuertas@ual.es*

^b*Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, Barcelona, Spain, carmen.hernando@upc.es*

^c*Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Barcelona, Spain, ignacio.m.pelayo@upc.es*

^d*Departament de Matemàtica Aplicada II, Universitat Politècnica de Catalunya, Barcelona, Spain, merce.mora@upc.es, carlos.seara@upc.es*

Abstract

Let G be a finite simple connected graph. A vertex v is a boundary vertex of G if there exists a vertex u such that no neighbor of v is further away from u than v . We obtain a number of properties involving different types of boundary vertices: peripheral, contour and eccentric vertices. Before showing that one of the main results in the paper "Boundary vertices in graphs" (by G. Chartrand and Alt, Disc. Math. 263 (2003) 25-34) does not hold for one of the cases, we establish a realization theorem that not only corrects the mentioned wrong statement but also improves it.

Given $S \subseteq V(G)$, its geodetic closure $I[S]$ is the set of all vertices lying on some shortest path joining two vertices of S . We prove that the boundary vertex set $\partial(G)$ of any graph G is geodetic, that is, $I[\partial(G)] = V(G)$. A vertex v belongs to the contour $Ct(G)$ of G if no neighbor of v has an eccentricity greater than v . We present some sufficient conditions to guarantee the geodeticity of either the contour $Ct(G)$ or its geodetic closure $I[Ct(G)]$.

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² Corresponding author. e-mail: carlos.seara@upc.es