

On the connectivity of cages with girth five, six and eight

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Abstract

A (δ, g) -cage is a regular graph of degree δ and girth g with the least possible number of vertices. Recently, some authors have addressed the problem of studying their connectivity parameters. In this direction, it was conjectured by Fu, Huang and Rodger that every (δ, g) -cage is maximally connected, i.e., it is δ -connected, and they proved this statement for $\delta = 3$. We provide a new contribution to the proof of that conjecture, by showing that every (δ, g) -cage, with $g = 6, 8$, is maximally connected, and by assuring either maximal connectivity or superconnectivity for some $(\delta, 5)$ -cages.

Key words. cage, connectivity, cutset.

MSC 2000. 05C35, 05C40.

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