On the geodetic and the hull numbers in strong product graphs☆

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Abstract

A set S of vertices of a connected graph G is convex, if for any pair of vertices $u, v \in S$, every shortest path joining u and v is contained in S. The convex hull $CH(S)$ of a set of vertices S is defined as the smallest convex set in G containing S. The set S is geodetic, if every vertex of G lies on some shortest path joining two vertices in S, and it is said to be a hull set if its convex hull is $V(G)$. The geodetic and the hull numbers of G are the cardinality of a minimum geodetic and a minimum hull set, respectively. In this work, we investigate the behavior of both geodetic and hull sets with respect to the strong product operation for graphs. We also stablish some bounds for the geodetic number and the hull number and obtain the exact value of these parameters for a number of strong product graphs.

Keywords: Metric graph theory, Geodetic set, Hull set, Geodetic number, Hull number, Strong product

1. Introduction

The process of rebuilding a network modelled by a connected graph is a discrete optimization problem, consisting in finding a subset of vertices of cardinalilty as small as possible, which, roughly speaking, would allow us to store and retrieve the whole graph. One way to approach this problem is by using a certain convex operator. This procedure has attracted much attention since it was shown by Farber an Jamison [13] that every convex subset in a graph is the convex hull of

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