THE LONGEST PATH TRANSIT FUNCTION OF A GRAPH AND BETWEENNESS

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Abstract. A longest path between two vertices in a connected graph G is a path of maximum length between the vertices. The longest path transit function $L(u, v)$ in a graph consists of the set of all vertices lying on any longest path between vertices u and v. A transit function L satisfies betweenness if $w \in L(u, v)$ implies $u \notin L(w, v)$ [(b1)-axiom] and $x \in L(u, v)$ implies $L(u, x) \subseteq L(u, v)$ [(b2)-axiom] and it is monotone if $x, y \in L(u, v)$ implies $L(x, y) \subseteq L(u, v)$. The betweenness and monotone axioms are discussed for the longest path transit function of G. The graphs for which the longest path transit function is a single path transit function are characterized.

Keywords: longest path transit function,convexity, betweenness, single path transit function

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1. INTRODUCTION

The notion of transit function is introduced in [16] as a means to study how to move around in discrete structures. It is a set function defined for every pair of points on a set V, which is provided with a structure σ, satisfying three simple axioms. Important examples of such a structure are the set of edges in a graph $G = (V, E)$, or a partial ordering $\leq$, on a partially ordered set $(V, \leq)$. Then transit functions are studied that have additional properties defined in terms of the structure σ. For example, the transit function may be defined in terms of paths in the graph $G = (V, E)$. Such transit functions are called *path transit functions* on G, a path function for short. Well studied path transit functions are the geodesic, induced and all-paths transit functions. Any transit function on (V, σ) defines a natural convexity on V. These three transit functions and their convexities are extensively studied; some relevant references are: for the geodesic transit function [12, 15, 17, 18, 23] for the induced-path transit function [11, 14] and for the all-paths transit function [2, 10, 19]. Path transit functions and their associated convexities are surveyed in [5]. See [1], for various notions of boundary sets using the geodesic convexity. It is interesting to note that the notions of transit functions and betweenness have found applications in the recombination theory in evolution, see for example, the papers of Stadler et. al. [21, 22, 24, 25]
It is a well-known fact that the graph distance $d(u, v)$ from a vertex u to a vertex v in a connected simple graph G, namely the length of a shortest u, v-path in G is a metric on G and it is an important tool for dealing with various distance notions in G. Similarly, the length $D(u, v)$ of a longest u, v-path in G namely the *detour distance* is also a metric that has caught attention among graph theorists recently. Many analogous notions of the distance function $d(u, v)$ have appeared in the literature using the detour distance $D(u, v)$;