Diameter vulnerability of GC graphs

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Abstract

Concern over fault tolerance in the design of interconnection networks has stimulated interest in finding large graphs with maximum degree Δ and diameter D such that the subgraphs obtained by deleting any set of s vertices have diameter at most D', this value being close to D or even equal to it. This is the so-called (Δ, D, D', s)-problem. The purpose of this work has been to study this problem for $s = 1$ on some families of generalized compound graphs. These graphs were designed by one of the authors in [15] as a contribution to the (Δ, D)-problem, that is, to the construction of graphs having maximum degree Δ, diameter D and an order large enough. When approaching the mentioned problem in these graphs, we realized that each of them could be redefined as a compound graph, the main graph being the underlying graph of a certain iterated line digraph. In fact, this new characterization has been the key point to prove in a suitable way that the graphs belonging to these families are solutions to the $(\Delta, D, D + 1, 1)$-problem.

MSC: 05C40, 05C12, 05C20

Keywords: Compound graph; (Δ, D, D', s)-problem; Graph on alphabets; Large graph; Vulnerability

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