On monophonic sets in graphs

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We deal with two types of graph convexities, which are defined by a system $\mathcal{P}$ of paths in a connected graph $G = (V, E)$: the geodetic convexity (also called the metric convexity)[3, 4] which arises when we consider shortest paths, and the monophonic convexity (also called the minimal path convexity)[2, 3] when we consider chordless paths. Given G and two vertices u, v in V, a chordless $u - v$ path in G is called a $u - v$ monophonic path. Let $J[u, v]$ denote the set of all vertices in G lying on some $u - v$ monophonic path. Given a set $S \subseteq V$, let $J[S] = \bigcup_{u,v \in S} J[u, v]$. If $J[S] = V$, then S is called a monophonic set of G. If $J[S] = S$, then S is called a m-convex set of G. The monophonic convex hull $[S]_m$ of S is the smallest m-convex set containing S. If $[S]_m = V$, then S is called a m-hull set of G. If we restrict ourselves to shortest paths, we obtain the geodetic and g-hull sets, which have been widely studied in the recent years.

We study monophonic sets in a connected graph G. Firstly, we present a realization theorem proving that there is no general relationship between monophonic and geodetic hull sets. Second, we study the contour of a graph [1] (a generalization of the set of extreme vertices) showing that the contour of G is a monophonic set. Finally, we focus our attention on the edge Steiner sets. We prove that every edge Steiner set S in G is edge monophonic, i.e., every edge of G lies on some monophonic path joining two vertices of S.

References.

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