On the Steiner set problem in graphs

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(joint work with C. Hernando, T. Jiang, M. Mora, I. M. Pelayo)

The most natural convexities on graphs are path convexities defined by a system $\mathcal{P}$ of paths in a graph G that contains all geodesics. The canonical choices for $\mathcal{P}$ are provided by selecting all paths, triangle paths, monophonic paths and geodesics.

The intrinsic or standard convexity is the so-called geodesic convexity in which a vertex set S is said to be (geodesically) convex if $S = I[S]$, where $I[S]$ (called the geodetic closure of S) is the set of all shortest $x - y$ paths (also called geodesics) for every $x, y \in S$. A set of vertices is called geodesic if $I[S] = V$, and it is said to be a hull set if $[S]_g = V$, where $[S]_g$ (called the convex hull of S) is the smallest convex set containing S. Certainly, every geodetic set is a hull set.

Another kind of convexity is the monophonic convexity in which a vertex set S is said to be monophonically convex if $S = J[S]$, where $J[S]$ (called the monophonic closure of S) is the set of all chordless $x - y$ paths (also called monophonic path) for every $x, y \in S$. A set of vertices is called monophonic if $J[S] = V$. Certainly, every monophonic set is a geodetic set.

Given a vertex set W, a Steiner W-tree is a tree containing W having minimum order. Moreover, W is called a Steiner set if $S(W) = V$, where $S(W)$ (called the Steiner interval of W) is the collection of all vertices of G which lies on some Steiner W-tree.

Contrarily to what was stated in a recent paper [1], Pelayo proved in [2] that not every Steiner set is a geodesic set. Going one step further, we approached the question: Is every Steiner set a monophonic set? obtaining an affirmative answer.

References

[1] G. Chartrand and P. Zhang, The Steiner number of a graph, *Disc. Math.*, **242** (2002), 41–54.

[2] I. M. Pelayo, Not every Steiner set is geodetic (a note on "The Steiner number of a graph"), *Disc. Math.*, submmited.