3.5 Consideramos la función $f(x) = \dfrac{2x^2+4}{x^2+2x+5}$.

(a) Resuelve la integral $\int f(x)dx$.

(b) Calcula la primitiva F de la función f que cumple la condición $F(-1) = -2$.

SOLUCIÓN:

(a) Dividimos: $\dfrac{2x^2+4}{x^2+2x+5} = 2 - \dfrac{4x+6}{x^2+2x+5}$

$$I = \int \frac{2x^2+4}{x^2+2x+5}\mathrm{d}x = \int 2\mathrm{d}x \text{ - } \int \frac{4x+6}{x^2+2x+5}\mathrm{d}x = 2x - I_2$$

$$x^2+2x+5=0 \Longrightarrow x = \frac{-2 \mp \sqrt{4-20}}{2} = \begin{cases} -1-2i \\ -1+2i \end{cases} \Longrightarrow x^2+2x+5 = (x+1)^2+4$$

Cambio de variable para la integral I_2: $x+1 = 2t \Longrightarrow \mathrm{d}x = 2\mathrm{d}t$

$$I_2 = \int \frac{4x+6}{x^2+2x+5}\mathrm{d}x = \int \frac{4(2t-1)+6}{4t^2+4}2\mathrm{d}t = \int \frac{4t+1}{t^2+1}\mathrm{d}t = \int \frac{4t}{t^2+1}\mathrm{d}t + \int \frac{1}{t^2+1}\mathrm{d}t =$$

$$= 2\ln(t^2+1) + \arctan t + K = 2\ln\frac{x^2+2x+5}{4} + \arctan\frac{x+1}{2} + K$$

$$I = 2x - I_2 \Longrightarrow \boxed{I = 2x - 2\ln\frac{x^2+2x+5}{4} - \arctan\frac{x+1}{2} + K}$$

(b) $F(-1) = -2 - 2\ln 1 - \arctan 0 + K = -2 \Longrightarrow K = 0 \Longrightarrow \boxed{F(x) = 2x - 2\ln\dfrac{x^2+2x+5}{4} - \arctan\dfrac{x+1}{2}}$