
3.4 Consideramos la función $f(x) = \frac{4x+6}{x^2+4}$.

(a) Resuelve la integral $\int f(x)dx$.

(b) Calcula la primitiva F de la función f que cumple la condición $F(0) = \ln 16$.

SOLUCIÓN:

(a) $\frac{4x+6}{x^2+4} = \frac{4x}{x^2+4} + \frac{6}{x^2+4}$

$$I = \int \frac{4x+6}{x^2+4} dx = \int \frac{4x}{x^2+4} dx + \int \frac{6}{x^2+4} dx = I_1 + I_2$$

$$I_1 = \int \frac{4x}{x^2+4} dx = 2 \int \frac{2x}{x^2+4} dx = 2 \ln(x^2+4) + K_1$$

Cambio de variable para la integral I_2 : $x = 2t \implies dx = 2dt$

$$I_2 = \int \frac{6}{x^2+4} dx = \int \frac{6}{4t^2+4} 2dt = 3 \int \frac{1}{t^2+1} dt = 3 \arctan \frac{x}{2} + K_2$$

$$I = I_1 + I_2 \implies I = 2 \ln(x^2+4) + 3 \arctan \frac{x}{2} + K$$

(b) $F(0) = 2 \ln 4 + K = \ln 16 \implies K = 0 \implies F(x) = 2 \ln(x^2+4) + 3 \arctan \frac{x}{2}$
