Sixty Years of the Degree-Diameter Problem.

Francesc Comellas

Department of Mathematics
Universitat Politècnica de Catalunya - Barcelona Tech

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Overview

- 60's - 70's – Influential papers from Hoffman and Singleton, and Elspas leave to a particular interest in the degree-diameter problem.
- 80's - 90's – Intense activity at LRI Paris - Delorme, Bermond, Farhi, ..– with connections to Belgium –Quisquater, Buset, ..– and UPC in Barcelona –Gómez, Fiol, Yebra, FC ..– and also in New Zealand –Hafner, Dinneen– and Germany –Sampels–. All this work lead to lasting collaborations and results.
- January 1995 at UPC first online table of the problem
- At the turn of the century, new approaches emerged from US –Exoo– and Oceania –Loz, Siran, Miller, Pineda-Villavicencio, Perez-Roses, ...– with significant improvements.
- 2005 – Relevant milestones: review paper by Miller and Siran (2005) and new web tables at combinatoricswiki.org.
- Past fifteen years, progress limited basically to a few hard-to-find small order graphs.

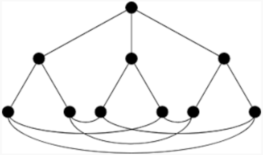
Moore graphs and the (Δ, D) problem

Moore bound and Moore graphs.

Hoffman and Singleton introduced the concept of Moore graphs, after Edward.Forrest Moore, and proved that

The largest possible order of a graph with maximum degree Δ and diameter D, $n(\Delta, D)$, is bounded by

$$n(\Delta, D) \leq 1 + \Delta + \Delta(\Delta-1) + \cdots + \Delta(\Delta-1)^{D-1}$$
$$\begin{cases} = \frac{\Delta(\Delta-1)^D - 2}{\Delta - 2}, & \text{if } \Delta > 2 \\ = 2D+1, & \text{if } \Delta = 2 \end{cases}$$



This value is called the *Moore bound*, and a graph attaining it is known as *Moore graph*. H-S proved that for $D \geq 2$ and $\Delta \geq 3$, this bound is only attained if $D = 2$ and $\Delta = 3, 7$, and (perhaps) 57).

- A. J. Hoffman and R. R. Singleton, "On Moore Graphs with Diameters Two and Three," I.B.M. Jour., 4, pp. 497-504 (November 1960)..

1964. First (Δ,D) table ?.

Bernard Elspas, in his 1964 paper:
A Moore graph is regular of degree Δ and it may be visualized as a D-level tree with suitable interconnections between the tip nodes. As might be expected, Moore graphs exist only for certain values of (Δ, D); they are, in fact, quite rare.

THE FUNCTION n(d,k)

d	k = 1	2	3	4	5	6	7
1	2	--	--	--	--	--	--
2	3	5	7	9	11	13	15
3	4	10	20	28	36	44	60
			(22)	(46)	(94)	(190)	(382)
4	5	15	27				
		(17)	(53)	(161)	(485)	(1457)	
5	6	24	36	60			
		(26)	(106)	(426)	(2230)		
6	7	35?					
		(37)	(187)	(937)			
7	8	50	78				
			(302)				

Key: Circled entries are maximal; underlined entries meet the Moore bound, $n_M(d,k)$; values in parentheses are of $n_M(d,k)$.

In red, values remaining in the current table.

- Bernard Elspas. Topological constraints in interconnection-limited logic. Proceedings of the Fifth Annual Symposium on Switching Circuit Theory and Logical Design, I.E.E.E. Publication S-164 (1964), pp. 133-1471.

70's - 80's tables. The beginnings.

The first "complete" (Δ,D) table from R. M. Storwick (December 1970)

d \ k	1	2	3	4	5	6	7	8	9	10
1	2	—	—	—	—	—	—	—	—	—
2	3	5	7	9	11	13	15	17	19	21
3	4	10	20 (22)	28 (46)	36 (94)	60 M (190)	66 NH (2, 1) (382)	90 F (766)	138 NH (2, 1) (1534)	216 NB (2, 1) (3070)
4	5	15 (17)	35 A (53)	40 (161)	62 NH (3, 1) (485)	114 NB (3, 1) (1457)	188 NH (3, 1) (4373)	320 F (13121)	566 NH (3, 1) (39365)	996 NB (3, 1) (118097)
5	6	24 (26)	36 (106)	126 A (426)	120 (1706)	232 NB (4, 1) (6826)	442 NH (3, 1) (2 7306)	850 F (109226)	1770 NH (3, 1) (436906)	3512 NB (3, 1) (1747626)
6	7	31 (37)	55 (187)	105 A (937)	462 A (4687)	447 NB (4, 1) (23437)	867 NH (4, 1) (117187)	1872 F (585937)	4317 NH (4, 1) (2929687)	9465 NB (4, 1) (14648437)
7	8	50	80 (302)	150 (1814)	378 A (10886)	1716 A (65318)	1574 NH (4, 1) (391910)	3626 F (3351462)	9422 NH (4, 1) (14108774)	22836 NB (4, 1) (84652646)
8	9	57 (65)	105 (457)	175 (3201)	504 (22409)	1386 A (156865)	6435 A (1098057)	6400 F (7686401)	18076 NH (5, 1) (53804809)	47880 NB (5, 1) (376633665)
9	10	74 (82)	150 (658)	240 (5266)	666 (42130)	1904 NB (6, 1) (337042)	5148 A (2696338)	24310 A (21570706)	32706 NH (5, 1) (172565648)	94416 NB (5, 1) (1380525202)
10	11	91 (101)	200 (911)	320 (8201)	910 (73811)	2780 NB (7, 1) (664301)	6864 A (5978711)	19305 A (53808401)	92378 A (484275611)	170685 NB (6, 1) (4358480501)

Note: Underlined values are Moore graphs, circled values are maximal graphs. Those marked A are due to Akers, [2]; F, Friedman, [3]; M, Mager, [5]; NB, nonidentical blocking; NH, nonidentical hinging. All other values are from [1].

- Robert M. Storwick, Improved Construction Techniques for (d, k) Graphs, IEEE Trans. on Computers (1970) 1214?1216..

The (Δ,D) table from Bermond, Delorme and Quisquater (February 1982)

Largest known (Δ, D)-graphs (February 1982)

Δ \ D	2	3	4	5	6	7	8	9	10
3	P 10	$C_5 \times 4$ 20	LFQSU 34	AL 56	H_2r 128	H_2idr 158	Y 244	Y 340	Y 536
4	$K_3 \times 5$ 15	$P \times 4$ 40	$C_5 \times 19$ 95	H'_3 364	H_3r 731	H_3idr 837	BDQ 1400	$C_{12} \times C_{161}$ 1932	$C_8(5, 9)$ 2560
5	$K_3 \times 8$ 24	15×4 60	Q_4r 174	H'_3d 532	H_4r 2734	H_4idr 2988	$O_{2,4}$dr 5004	BDQ 11340	BDQ 30240
6	$K_4 \times 8$ 32	21×5 105	Q_5r 317	H'_3d 756	H_5r 7817	$H_4 \times 4$ 10920	$H_4 \times 6$r 16385	BDQ 43744	BDQ 131232
7	HS 50	24×5 120	Q_5dr 352	$Q_5 \times 4$ 1248	H_5dr 8998	$H_5 \times 4$ 31248	$24[P'_{47}]$ 54168	BDQ 156340	BDQ 562824
8	P'_7 57	$HS \times 4$ 200	Q_7r 807	$HS[K_{51}]$ 2550	H_7r 39223	H_7idr 40593	BDQ 154800	$H_5 \times 35$ 273420	$C_8(5, 9)$ 1310720
9	P'_8d 74	Q'_8 585	Q_8r 1178	$HS[K_{101}]$ 5050	H_8r 74906	$H_7 \times 4$ 156864	$HS[P'_{97}d]$ 480250	BDQ 1176690	BDQ 5883450
10	P'_9 91	Q'_8d 650	BW 1755	$HS[K_{151}]$ 7550	H_9r 132869	$Q'_8[K_{651}]$ 380835	$HS[P'_{149}]$ 1117550	BDQ 2696616	BDQ 14981200
11	P'_9d 94	Q'_8d 715	$Q'_8 \times 5$ 2925	$P'_8[K_{156}]$ 11388	H_9dr 142494	$Q'_8[K_{1236}]$ 723060	$HS[P'_{199}]$ 1990050	BDQ 5580498	BDQ 33217250
12	P'_{11} 133	Q'_8d 780	$Q'_8 \times 8$ 4680	$P'_9[K_{193}]$ 17563	H_{11}r 354323	$Q'_8[K_{1821}]$ 1065285	$P'_8[P'_{227}]$ 3778261	LVLQ 10077696	$C_p(3,5)$ 85887453
13	P'_{11}d 136	Q'_8d 845	$Q'_8 \times 9$ 5265	$P'_9[K_{284}]$ 25844	H_{11}dr 394616	$715[K_{2016}]$ 1414440	$P'_9[P'_{281}d]$ 7211386	$4680[K_{5201}]$ 24340680	BDQ 121296802
14	P'_{13} 183	Q'_8d 910	650×9 5850	$P'_{11}[K_{279}]$ 37107	H_{13}r 804481	$910[K_{2341}]$ 2130310	$P'_9[P'_{373}]$ 12694773	$4680[K_{9881}]$ 46243080	$C_8(1, 10)$ 282475249
15	P'_{13}d 186	D 1215	$Q'_8 \times 13$ 7605	$P'_{11}[K_{412}]$ 54796	H_{13}dr 892062	D 5133375	$P'_{11}[P'_{409}d]$ 22303302	$4680[K_{14561}]$ 68145480	BDQ 447391446

- J.-C. Bermond, C. Delorme and J.-J. Quisquater, Tables of large graphs with given degree and diameter, Inform. Process. Lett. 15 (1982) 10-13

Quisquater: connection Belgium - Paris - Barcelona

Mathematics Genealogy Project

Jean-Jacques Quisquater

MathSciNet

D.Sc. Université Paris-Sud XI - Orsay 1987

Dissertation: *Structures d'interconnexion*

Mathematics Subject Classification: 68—Computer science

Advisor 1: Jean-Claude Bermond

Advisor 2: José Luis Andrés Yebra

Jean-Jacques Quisquater

3 languages

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From Wikipedia, the free encyclopedia

Jean-Jacques Quisquater (born 13 January 1945) is a Belgian cryptographer and a professor at University of Louvain (UCLouvain). He received, with Claus P. Schnorr, the RSA Award for Excellence in Mathematics in 2013, and the ESORICS Outstanding Research Award 2013.[1]

On Saturday, 1 February 2014, Flemish public news agency VRT reported that about 6 months earlier, Quisquater's personal computer had been hacked.[2] Since the same hacking technique was used at Belgium's public/private telecom provider Belgacom, VRT makes links to the NSA hacking scandal. Still according to VRT, a week before the article went out Edward Snowden warned about the NSA also targeting companies and private persons, in an interview with German television channel ARD. Belgian newspaper De Standaard mentions GCHQ and says the authorities are investigating the case.[3] Reporters write Quisquater's computer was infected with malware after clicking a bogus invitation to join a social network —"that allowed the intruders to follow all of the professor's digital movements, including his work for international conferences on security".

Jean-Jacques Quisquater

Born	13 January 1945 (age 78)
Alma mater	University of Paris-Sud
	Scientific career
Institutions	Philips University of Louvain
Thesis	*Structures d'interconnexion: Constructions et applications* (1987)
Doctoral advisor	Jean-Claude Bermond

References [edit]

1. ^ "ESORICS Home Page".
2. ^ "Belgische hackingexpert nu zelf slachtoffer". February 2014.
3. ^ "ENGLISH SUMMARY – Belgian professor in cryptography hacked".

1983-1986. Construction for $\Delta = 3, D = 4, n = 38$ (optimal)

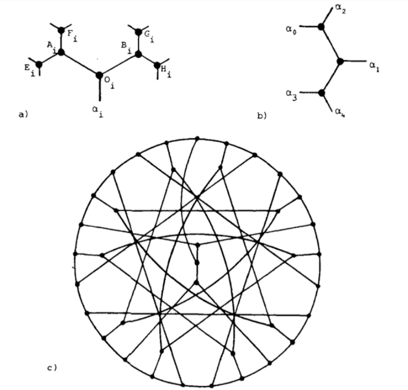


FIG. 5

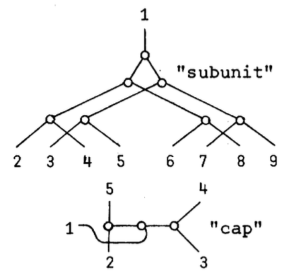
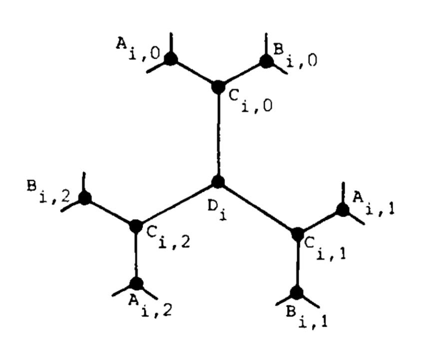


Fig. 2. Parts of SP (38, 3, 4).

- C. Von Conta, Torus and other networks as communication networks with up to some hundred points, IEEE Transactions on Computers 32 (7) (1983) 657-666.
- I. Alegre, M.A. Fiol and J.L.A. Yebra, Some large graphs with given degree and diameter, J. Graph Theory 10 (1986) 219 - 224.
- D. Buset, Maximal cubic graphs with diameter 4, Discrete Applied Mathematics 101 (1-3) (2000) 53- 61.

1983-1986. Construction for $\Delta = 3, D = 5, n = 70$

Connect seven identical clusters of 10 vertices according to $A_{i,j} \equiv B_{i \pm 2^j, j+1}$



- C. Von Conta, Torus and other networks as communication networks with up to some hundred points, IEEE Transactions on Computers 32 (7) (1983) 657-666.
- I. Alegre, M.A. Fiol and J.L.A. Yebra, Some large graphs with given degree and diameter, J. Graph Theory 10 (1986) 219?224.

1985. Compound graphs from Gómez and Fiol

D / Δ	2	3	4	5	6	7	8	9	10
9	P'_8d 74	Q'_8 585	** $Q_5(T_4)$ 1 248	* $HS]C_{103}[$ 5 150	H_8r 74 906	* $H_7\Lambda pt$ 215 688	* $H_7\Lambda K_5$ 588 240	* $H_7\Lambda HS$ 2 941 200	** $Q_7\nabla_1H_7$ 15 686 400
10	P'_9 91	Q'_8d 650	$(Q_8(T_3))'$ 1 755	** $K_1\Sigma_3Q_7$ 8 400	H_9r 132 869	* $H_8\Lambda pt$ 486 837	* $H_8\Lambda K_6$ 1 348 164	* $H_8\Lambda HS$ 7 489 800	** $Q_7\Sigma_2H_7$ 47 059 200
11	P'_9d 94	Q'_8d 715	** $Q_7(T_4)$ 3 200	** $K_1\Sigma_3Q_8$ 12 285	** $H_7(T_4)$ 156 864	* $H_9\Lambda pt$ 863 590	* $H_9\Lambda K_6$ 2 790 060	** $HS\Sigma_3H_8$ 16 852 050	* $H_9\Lambda Q'_8$ 120 969 030
12	P'_{11} 133	Q'_8d 780	$Q'_8 * K_8$ 4 680	** $K_1\Sigma_3Q_9$ 18 860	$H_{11}r$ 354 323	** $K_1\Sigma_3H_9$ 1 527 890	** $K_{6,6}\Sigma_2H_9$ 4 782 960	** $P_9\Sigma_2H_9$ 36 270 780	** $Q_9\Sigma_2H_9$ 326 835 600
13	$P'_{11}d$ 136	Q'_8d 845	** $Q_9(T_4)$ 6 560	$Q'_8 * mP'_7$ 33 345	** $H_9(T_4)$ 531 440	* $H_{11}\Lambda pt$ 2 657 340	* $H_{11}\Lambda K_7$ 9 920 736	** $P_9\Sigma_1H_9$ 72 541 560	** $Q_9\nabla_1H_{11}$ 581 071 680
14	P'_{13} 183	Q'_8d 910	** $Q_9(T_5)$ 8 200	$Q'_8 * mP'_8$ 42 705	$H_{13}r$ 804 481	** $K_1\Sigma_3H_{11}$ 4 783 212	** $K_7\Sigma_3H_{11}$ 18 601 380	** $P'_9\Sigma_3H_{11}$ 145 090 764	** $Q_{11}\Sigma_2H_{11}$ 1 556 138 304
15	$P'_{13}d$ 186	$(*Q_{2,4})'$ 1 215	** $Q_{11}(T_4)$ 11 712	* $P'_{11}]C_{414}[$ 55 062	** $H_{11}(T_4)$ 1 417 248	* $H_{13}\Lambda pt$ 6 837 978	* $H_{13}\Lambda K_6$ 28 960 848	** $P_{11}\Sigma_1H_{11}$ 282 740 976	** $Q_{11}\nabla_1H_{13}$ 2 355 482 304
16	$P'_{13}d$ 197	$(*Q_3)'$ 1 600	** $Q_{11}(T_5)$ 14 640	$(*H_3)'$ 132 496	** $H_{11}(T_5)$ 1 771 560	** $K_1\Sigma_3H_{13}$ 11 664 786	** $K_9\Sigma_3H_{13}$ 54 301 590	** $P_{11}\Sigma_2H_{13}$ 481 474 098	** $Q_{13}\Sigma_2H_{13}$ 5 743 901 520

Table of largest known (Δ,D) graphs (July 1985).

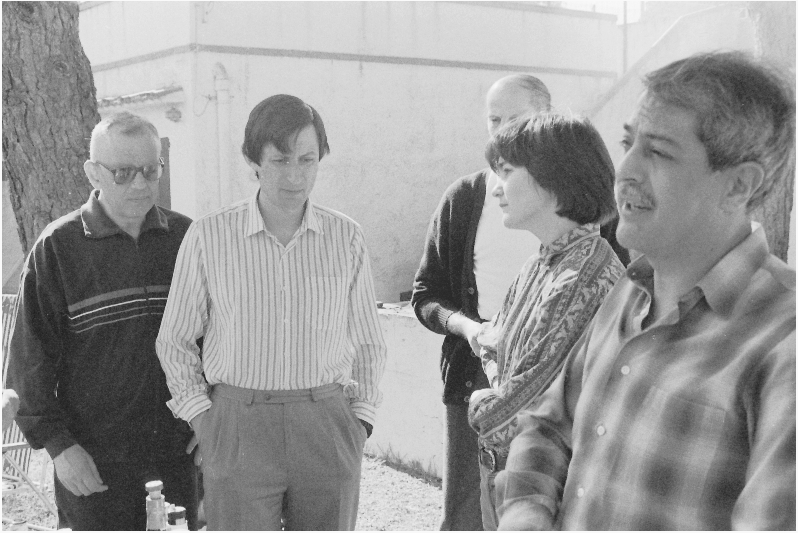
- J. Gomez, M.A. Fiol. Dense Compound Graphs. Ars Combinatoria, 20-A (1985), pp. 211-237

1986. Gómez PhD. Workshop at Luminy, (Barcelona - Paris).

- José Gómez-Martí, Diámetro y Vulnerabilidad en Redes de Interconexión. PhD Thesis, UPC, Barcelona 1986. Supervisor: M.A. Fiol

90's tables. Progress.

1987-1990 Intense work at Barcelona and Paris. Barbecue "Chez Yebra" Feb. 3rd, 1990



- The Degree-Diameter Problem, D.I. Ameter, Max Degree,

1987-1990 Intense work at Barcelona and Paris. At the beach

- The Degree-Diameter Problem, D.I. Ameter, Max Degree,

1987-1990 Intense work at Barcelona and Paris.

- The Degree-Diameter Problem, D.I. Ameter, Max Degree,

1989. More compound graphs

Largest known (Δ, D) graphs (January 1989) (the new data obtained in this paper are represented in boldface)

Δ \ D	2	3	4	5	6	7	8	9	10
3	P 10	$C_5 * F_4$ 20	YFA 38	YFA 70	**H_2t** **130**	CR^* 184	CR^* 320	$2cy$ 540	$2cy$ 938
4	$K_3 * C_5$ 15	$P^* F_4$ 40	$C_5 * C_{19}$ 95	H'_3 364	**H_3s** **734**	CCD 1081	CCD 2943	CCD 7439	CCD 15 657
5	$K_3 * X_8$ 24	$Lente$ 70	**Q_4s** **182**	H'_3d 532	**H_4s** **2742**	$8a$ 4368	$2cy$ 11 200	$2cy$ 33 600	$8a$ 123 120
6	$K_4 * X_8$ 32	$C_5 * C_{21}$ 105	$8a$ 355	$8a$ 1081	**H_5s** **7832**	$8a$ 13 104	$8a$ 50 616	$8a$ 202 464	$8a$ 682 080
7	HS 50	$Allwr$ 128	$(15) * m(32)$ 480	$8a$ 2162	**H_6s** **10 554**	$8a$ 39 732	$2cy$ 140 000	$8a$ 911 088	$2cy$ 2 002 000
8	P'_7 57	$8a$ 203	**Q_7s** **842**	$8a$ 2880	**H_7s** **39 258**	$8a$ 89 820	$8a$ 455 544	$8a$ 1 822 176	**$Q_4\Sigma_6H_5$** **3 984 120**
9	P'_8d 74	Q'_8 585	$Q_5(T_4)$ 1248	$8a$ 6072	**H_8s** **74 954**	$H_7 \wedge K_1$ 215 688	$2cy$ 910 000	**$HS \wedge_p H_7$** **3 019 632**	$Q_7\nabla_1H_7$ 15 686 400
10	P'_0 91	Q'_8d 650	**Q_9s** **1820**	$8a$ 12 144	**H_9s** **132 932**	$H_8 \wedge K_1$ 486 837	$2cy$ 2 002 000	**$HS \wedge_p H_8$** **7 714 494**	**$Q_7\Sigma_2H_7$** **47 059 200**
11	P'_0d 94	Q'_8d 715	$Q_7(T_4)$ 3200	**$K_1\Sigma_8Q_8$** **14 625**	$H_7(T_4)$ 156 864	**$K_{1,1}\Sigma'_7H_8$** **898 776**	**$K_{6,6}\Sigma_6H_8$** **4 044 492**	**$P_7\Sigma_7H_8$** **21 345 930**	**$Q_7\Sigma_6H_8$** **179 755 200**
12	P'_{11} 133	Q'_8d 780	$Q'_8 * X_8$ 4680	$8a$ 24 360	**$H_{11}s$** **354 422**	**$K_{1,1}\Sigma'_7H_9$** **1 727 180**	**$K_{7,7}\Sigma_6H_9$** **8 370 180**	**$P_8\Sigma_7H_9$** **48 493 900**	**$Q_8\Sigma_6H_9$** **466 338 600**
13	$P'_{11}d$ 136	Q'_8d 845	$Q_0(T_4)$ 6560	$Q'_8 * m(P'_7)$ 33 345	$H_9(T_4)$ 531 440	$H_{11} \wedge K_1$ 2 657 340	**$K_{1,3}\Pi' H_{11}$** **10 629 360**	$P_9\Sigma_1H_9$ 72 541 560	**$Q_9\Sigma'_6H_9$** **762 616 400**
14	P'_{13} 183	Q'_8d 910	$Q_0(T_5)$ 8200	**$K_1\Sigma_8Q_{11}$** **51 240**	**$H_{13}s$** **804 624**	**$K_1\Sigma_8H_{11}$** **6 200 460**	**$K_{7,7}\Sigma_6H_{11}$** **29 762 208**	**$P_9\Sigma_7H_{11}$** **164 755 080**	**$Q_8\Sigma_6H_{11}$** **1 865 452 680**
15	$P'_{13}d$ 186	$(\otimes Q_{2,4})'$ 1215	$Q_{11}(T_4)$ 11 712	**$K_1\Sigma'_8Q_{11}$** **58 560**	$H_{11}(T_4)$ 1 417 248	**$K_1\Sigma'_8H_{11}$** **7 086 240**	**$K_{8,8}\Sigma_6H_{11}d$** **35 947 392**	$P_{11}\Sigma_1H_{11}$ 282 740 976	**$Q_{11}\Sigma'_6H_{11}$** **3 630 989 376**
16	$P'_{13}d$ 197	$(\otimes Q_3)'$ 1600	$Q_{11}(T_5)$ 14 640	$(\otimes H_3)'$ 132 496	$H_{11}(T_5)$ 1 771 560	**$K_1\Sigma_8H_{13}$** **14 882 658**	**$K_{9,9}\Sigma_6H_{13}$** **86 882 544**	**$P_9\Sigma_7H_{11}$** **585 652 704**	**$Q_{11}d\Sigma_6H_{13}$** **7 394 669 856**

- J. Gómez, M.A. Fiol and O. Serra, On large (Δ, D)-graphs, Discrete Mathematics 114 (1993) 219-235.

1991-1992. Vertex replacement by K_3. $\Delta = 4, D = 6, n = 740$

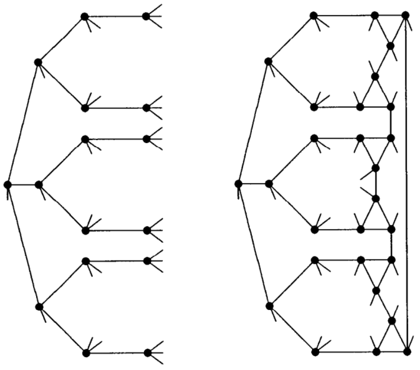


Figure 3: Modification of H_3 that gives $H_3(K_3)$

- F. Comellas and J. Gómez, New large graphs with given degree and diameter, Graph Theory, Combinatorics and Algorithms 1,2: Proc. 7th Quadrennial Int'l Conf. on the theory and Appl. of Graphs, Kalamazoo (MI, USA) (1992), edited by Y. Alavi and A. Schwenk (1995) 221?233

1992. Kalamazoo. 7th Quad. Int. Conf. Theory and Appl. of Graphs

F. COMELLAS AND J. GÓMEZ

Δ \ D	2	3	4	5	6	7	8
3	P 10	$C_5 * F_4$ 20	vC 38	vC 70	GFS 130	CR^* 184	CR^* 320
4	$K_3 * C_5$ 15	$Allwr$ 41	$C_5 * C_{19}$ 95	H'_3 364	$H_3(K_3)$ **740**	DH 1 155	DH^{**} 3 025
5	$K_3 * X_8$ 24	$Lente$ 70	$Q_4(K_3)$ **186**	H'_3d 532	$H_4(K_4)$ **2 754**	DH 5 334	DH 15 532
6	$K_4 * X_8$ 32	$C_5 * C_{21}$ 105	DH^* 360	DH 1 230	$H_5(K_4)$ **7 860**	DH 18 775	DH 69 540
7	HS 50	DH^* 144	DH^* 600	DH 2 756	$H_4(K_4){<}H_5$ **10 566**	DH 47 304	DH 214 500
8	P'_7 57	DH 234	DH 1 012	DH^* 4 704	$H_7(K_6)$ **39 396**	DH 127 134	DH 654 696
9	P'_8d 74	Q'_8 585	DH 1 430	DH 7 344	$H_8(K_6)$ **75 198**	DH 264 024	DH^{**} 1 354 896
10	P'_9 91	Q'_8d 650	DH 2 200	DH^* 12 288	$H_9(K_6)$ **133 500**	DH 554 580	DH^{**} 3 069 504
11	P'_9d 94	Q'_8d 715	$Q_7(T_4)$ 3 200	DH 17 458	$H_7(T_4)$ 156 864	DH 945 574	Cam 4 773 696
12	P'_{11} 133	Q'_8d 780	$Q'_8 * X_8$ 4 680	DH 26 871	$H_{11}(K_6)$ **355 812**	Din 1 732 514	DH 10 007 820
13	$P'_{11}d$ 136	Q'_8d 845	$Q_9(T_4)$ 6 560	DH 37 056	$H_9(T_4)$ 531 440	Cam 2 723 040	DH 15 027 252
14	P'_{13} 183	Q'_8d 910	$Q_9(T_5)$ 8 200	DH 53 955	$H_{13}(K_7)$ **806 636**	$K_1\Sigma_8H_{11}$ 6 200 460	Din 29 992 052

- F. Comellas and J. Gómez, New large graphs with given degree and diameter, Proc. 7th Quadrennial Int?l Conf. on the theory and Appl. of Graphs, Kalamazoo (MI, USA) (1992), edited by Y. Alavi and A. Schwenk (1995) 221?233.

1992. Kalamazoo. 7th Quad. Int. Conf. Theory and Appl. of Graphs

Do you recognize anyone in this photo??.

1993. Dinnen and Hafner. Semidirect product. Random search

Δ \ D	2	3	4	5	6	7	8	9	10
3	P 10	$C_5 * F_4$ 20	vC 38	vC 70	GFS 130	CR* 184	CR* 320	2cy 540	2cy 938
4	$K_3 * C_5$ 15	Allwr 41	$C_5 * C_{19}$ 95	H'_3 364	$H_3(K_3)$ 740	**DH** **1 155**	**DH**** **3 025**	**DH** **7 550**	**DH** **16 555**
5	$K_3 * X_8$ 24	Lente 70	$Q_4(K_3)$ 186	$H'_3\ d$ 532	$H_4(K_3)$ 2 754	**DH** **5 334**	**DH** **15 532**	**DH** **49 932**	**DH** **145 584**
6	$K_4 * X_8$ 32	$C_5 * C_{21}$ 105	**DH*** **360**	**DH** **1 230**	$H_5(K_4)$ 7 860	**DH** **18 775**	**DH** **69 540**	**DH** **275 540**	**DH** **945 574**
7	HS 50	**DH*** **144**	**DH˙** **600**	**DH** **2 756**	$H_4(K_4) < H_5$ 10 566	**DH** **47 304**	**DH** **214 500**	**DH** **945 574**	Cam 4 773 697
8	P'_7 57	**DH** **234**	**DH** **1 012**	**DH*** **4 704**	$H_7(K_6)$ 39 396	**DH** **127 134**	**DH** **654 696**	**DH**** **2 408 704**	Cam 7 738 848
9	$P'_8\ d$ 74	Q'_8 585	**DH** **1 430**	**DH** **7 344**	$H_8(K_6)$ 75 198	**DH** **264 024**	**DH**** **1 354 896**	**DH** **4 980 696**	Cam 19 845 936
10	P'_9 91	$Q'_8\ d$ 650	**DH** **2 200**	**DH*** **12 288**	$H_9(K_6)$ 133 500	**DH** **554 580**	**DH**** **3 069 504**	**DH** **9 003 000**	$Q_7\Sigma_2H_7$ 47 059 200
11	$P'_9\ d$ 94	$Q'_8\ d$ 715	$Q_7(T_4)$ 3 200	**DH** **17 458**	$H_7(T_4)$ 156 864	**DH** **945 574**	Cam 4 773 696	Cam 25 048 800	$Q_7\Sigma_6H_8$ 179 755 200
12	P'_{11} 133	$Q'_8\ d$ 780	$Q'_8 * X_8$ 4 680	**DH** **26 871**	$H_{11}(K_6)$ 355 812	Dinn 1 732 514	**DH** **10 007 820**	**DH** **48 532 122**	$Q_8\Sigma_6H_9$ 466 338 600
13	$P'_{11}\ d$ 136	$Q'_8\ d$ 845	$Q_9\ (T_4)$ 6 560	**DH** **37 056**	$H_9(T_4)$ 531 440	Cam 2 723 040	**DH** **15 027 252**	**DH** **72 598 920**	$Q_9\Sigma_6H_9$ 762 616 400
14	P'_{13} 183	$Q'_8\ d$ 910	$Q_9(T_5)$ 8 200	**DH** **53 955**	$H_{13}(K_7)$ 806 636	$K_1\Sigma_8H_{11}$ 6 200 460	Dinn 29 992 052	$P_9\Sigma_7H_{11}$ 164 755 080	$Q_8\Sigma_6H_{11}$ 1 865 452 680
15	$P'_{13}\ d$ 186	$(\otimes Q_{2,4})'$ 1 215	$Q_{11}(T_4)$ 11 712	**DH** **69 972**	$H_{11}(T_4)$ 1 417 248	**DH** **7 100 796**	**DH** **38 471 006**	$P_{11}\Sigma_7H_{11}$ 282 740 976	$Q_{11}\Sigma_6H_{11}$ 3 630 989 376
16	$P'_{13}\ d$ 197	$(\otimes Q_3)'$ 1 600	$Q_{11}(T_5)$ 14 640	$(\otimes H_3)'$ 132 496	$H_{11}(T_5)$ 1 771 560	$K_1\Sigma_8H_{13}$ 14 882 658	$K_{9,9}\Sigma_{61}H_{13}$ 86 882 544	$P_9\Sigma_7H_{11}$ 585 652 704	$Q_{11}\ d\Sigma_6H_{13}$ 7 394 669 856

- M.J. Dinneen, P.R. Hafner, New results for the degree/diameter problem, Networks 24 (1994) 359?367.

Paul Hafner (St John's day 2001, Catalan celebration)

1994. Semidirect product. Simulated anneling

The semidirect product of the cyclic groups Z_m with Z_n, when the multiplicative order of a unit A of Z_n divides m is defined by using the following multiplication rule:

for $x, u \in Z_n$ and $y, v \in Z_n$ the product is
$[x, y] \times [u, v] = [(x+u) \mod m, (y * A^u + v) \mod n]$.

Degree= **8**, Diameter = **3**; Order =**253**; Moore bound=457.
Obtained (08/1994) as a Cayley graph for semidirect product of Zm with Zn.

Group	Generators	Inverses
11*(9)23	[7 2]	[4 11]
	[10 4]	[1 10]
	[1 16]	[10 11]
	[9 17]	[2 3]
level 0	1	
level 1	8	
level 2	52	
level 3	192	

Comellas, F.; Mitjana M. (email aug.1994).
Download the adjacency list of the graph.

1995. First online (Δ,D) table.

LARGEST KNOWN (Δ,D)-GRAPHS (Feb. 95)

D/Δ	2	3	4	5	6	7	8	9	10
3	10	20	38	70	130	184	320	540	938
4	15	41	95	364	740	1.155	3.025	7.550	16.555
5	24	70	186	532	2.754	5.334	15.532	49.932	145.584
6	32	105	360	*1.260*	7.860	18.775	69.540	275.540	945.574
7	50	144	*630*	2.756	10.566	47.304	214.500	945.574	4.773.696
8	57	*253*	*1.081*	4.704	39.396	111.691	654.696	2.408.704	7.738.848
9	74	585	1.430	7.334	75.198	264.024	1.354.896	4.980.696	19.845.936
10	91	650	2.020	12.288	133.500	554.580	3.069.504	9.003.000	47.059.200

- http://maite71.upc.es/grup_de_grafs/table_g.html F. Comellas. UPC

1997. M. Sampels. Genetic algorithm

$\Delta \setminus D$	2	3	4	5	6	7	8	9	10
3	*10*	*20*	38	70	130	184	320	540	938
4	*15*	41	95	364	740	1 155	**3 080**	7 550	**17 604**
5	*24*	70	**210**	**546**	2 754	**5 500**	**16 956**	**52 768**	**145 880**
6	32	108	**375**	**1 395**	7 860	**19 065**	**74 256**	**278 046**	**954 480**
7	*50*	144	**672**	2 756	**11 110**	**50 020**	**216 160**	**953 586**	**5 243 030**
8	57	**253**	1 081	4 895	39 396	127 134	**660 765**	**2 943 720**	**7 739 472**
9	74	585	**1 536**	7 752	75 198	264 024	**1 355 424**	**5 094 726**	**19 873 350**
10	91	650	2 211	**12 642**	133 500	**556 803**	**3 696 600**	**9 910 080**	**47 129 712**

Fig. 4. Largest known graphs for a given degree Δ and diameter D (new results in **bold**, optimal results in *italics*)

- M. Sampels, (1997). Large networks with small diameter. WG 1997. Lect. Notes in Comput Sci. 1335 (1997) 288?302.

Michael Sampels, IWACOIN 99

2000's tables. US, Australia, New Zealand with important updates.

1998. Geoff Exoo. Relevant contribution to the (Δ,D) table.

LARGEST KNOWN (Δ,D)-GRAPHS (July 1998)

Δ\ D	2	3	4	5	6	7	8	9	10
3	10	20	38	70	132	190	330	570	950
4	15	41	96	364	740	1.155	3.080	7.550	17.604
5	24	72	210	552	2.760	5.500	16.956	53.020	164.700
6	32	110	380	1.395	7.908	19.279	74.800	294.679	1.211.971
7	50	148	672	2.756	11.220	52.404	233.664	1.085.580	5.243.030
8	57	253	1.081	5.050	39.671	129.473	713.539	4.039.649	13.964.808
9	74	585	1.536	7.884	75.696	270.048	1.485.466	8.911.766	25.006.478
10	91	650	2.211	12.788	134.395	561.949	4.019.489	13.964.808	52.029.411
11	98	715	3.200	18.632	156.864	970.410	5.211.606	48.626.760	179.755.200
12	133	780	4.680	29.435	358.183	1.900.319	10.007.820	97.386.380	466.338.600
13	162	845	6.560	39.402	531.440	2.901.294	15.733.122	145.880.280	762.616.400
14	183	912	8.200	56.325	812.924	6.200.460	29.992.052	194.639.900	1.865.452.680
15	186	1.215	11.712	73.984	1.417.248	7.100.796	45.000.618	282.740.976	3.630.989.376
16	198	1.600	14.640	132.496	1.771.560	14.882.658	86.882.544	585.652.704	7.394.669.856

- A family of graphs and the degree/diameter problem. J. Graph Theory 37 (2001), 118-124. Communicated May, 19-22, July, 1 1998

2002-2005. Mirka Miller and Joseph Širáň survey.

Moore graphs and beyond: A survey of the degree/diameter problem

Mirka Miller
School of Information Technology and Mathematical Sciences
University of Ballarat, Ballarat, Australia
mmiller@ballarat.edu.au

Jozef Širáň
Department of Mathematics
University of Auckland, Auckland, New Zealand
siran@math.auckland.ac.nz

Submitted: Dec 4, 2002; Accepted: Nov 18, 2005; Published: Dec 5, 2005
Mathematics Subject Classifications: 05C88, 05C89

- M. Miller, J. Širáň, Moore graphs and beyond: A survey of the degree/diameter problem, Electron J Combin DS14 (2005), 1?61.

2003 Newcastle

2006 PhD Eyal Loz

new degree diameter records

5 messages

eloz002@math.auckland.ac.nz <eloz002@math.auckland.ac.nz> Mon, Jul 3, 2006 at 9:06 AM
To: Charles Delorme <cd@lri.fr>, Francesc Comellas <comellas@ma4.upc.edu>
Cc: Eyal Loz <eyalloz@gmail.com>, Paul Bonnington <p.bonnington@auckland.ac.nz>, "siran@math.auckland.ac.nz" <siran@math.auckland.ac.nz>

Dear Charles and Francesc,
In the link below there are some Degree-Diameter record graphs in MAGMA format that were found as part of my ongoing PHD thesis study (supervised by Jozef Siran and Paul Bonnington), for the following degrees and diameters:
deg-6 diam-4 order-390
deg-8 diam-4 order-1100
deg-4 diam-7 order-1260
deg-4 diam-8 order-3243
deg-4 diam-9 order-7575
deg-4 diam-10 order-17703

http://www.math.auckland.ac.nz/~eloz002/degreediameter/

Many thanks,

Eyal Loz,
PHD student,
Auckland University Math department
New Zealand

Degree 4:
deg 4 diam 7 order 1260
deg 4 diam 8 order 3243
deg 4 diam 9 order 7575
deg 4 diam 10 order 17703
Degree 5:
deg 5 diam 5 order 624
deg 5 diam 7 order 5516
deg 5 diam 8 order 17030
Degree 6:
deg 6 diam 4 order 390
deg 6 diam 7 order 19282
Degree 8:
deg 8 diam 4 order 1100
deg 8 diam 5 order 5060
Degree 9:
deg 9 diam 5 order 8200
Degree 10:
deg 10 diam 4 order 2223

- Eyal Loz PhD. `http://www.math.auckland.ac.nz/∼eloz002/degreediameter/`

2006 PhD Eyal Loz Table

http://www.math.auckland.ac.nz/~eloz002/degreediameter/ Go DEC JAN 03 2006 2007
8 captures
3 Jan 2007 – 26 Feb 2021

Degree \ Diameter	4	5	6	7	8	9	10
4				1,320	3,243	7,575	17,703
5		624		5,516	17,030	53,352	164,720
6	390	1,404		19,282	75,157	295,025	1,212,117
7			11,988	52,768	233,700	1,124,990	5,311,572
8	1,100	5,060		130,017	714,010	4,039,704	17,823,532
9	1,550	8,200		270,192	1,485,498	10,423,212	31,466,244
10	2,223	13,140		561,957	4,019,736	17,304,400	104,058,822
11		18,700		971,028	5,941,864	62,932,488	250,108,668
12		29,470		1,900,464	10,423,212	104,058,822	600,105,100
13		39,576		2,901,404	17,823,532	180,002,472	1,050,104,118
14		56,790			41,894,424	450,103,771	2,050,103,984
15		74,298		8,079,298	90,001,236	900,207,542	
16					104,518,518	1,400,103,920	

Degree – Diameter Project:
Here is the table of the best known (D,D)-Graphs that were found as part of my ongoing PhD thesis study (supervised by Jozef Siran and Paul Bonnington). Adjacency lists for graphs of order less then 20,000 are linked from the table. The adjacency lists of the bigger graphs are available on demand.

The graphs above were made available for public viewing in July 2006.

Link to the online table of best known Degree-Diameter graphs.

I will make some indications of methods, techniques and theory in due time.

Thanks,

Eyal Loz.

- E. Loz, e-mail Jul 3, 2006 and web page.
  https://web.archive.org/web/20070103210426/http://www.math.auckland.ac.nz/∼eloz002/degreediameter/
- E. Loz, J. Širáň. New record graphs in the degree-diameter problem. Australas. J. Combin. 41 (2008), 63-80. (revised 3 Nov 2007)

2007-2008 Eyal Loz, J. Širáň. Table

$d\backslash k$	4	5	6	7	8	9	10
4				1,320	3,243	7,575	17,703
5		624		5,516	17,030	57,840	187,056
6	390	1,404		19,383	76,461	307,845	1,253,615
7			11,988	52,768	249,660	1,223,050	6,007,230
8	1,100	5,060		131,137	734,820	4,243,100	24,897,161
9	1,550	8,200		279,616	1,686,600	12,123,288	65,866,350
10	2,286	13,140		583,083	4,293,452	27,997,191	201,038,922
11		19,500		1,001,268	7,442,328	72,933,102	600,380,000
12		29,470		1,999,500	15,924,326	158,158,875	1,506,252,500
13		40,260		3,322,080	29,927,790	249,155,760	3,077,200,700
14		57,837			55,913,932	600,123,780	7,041,746,081
15		76,518		8,599,986	90,001,236	1,171,998,164	10,012,349,898
16					140,559,416	2,025,125,476	12,951,451,931

- E. Loz, J. Širáň. New record graphs in the degree-diameter problem. Australas. J. Combin. 41 (2008), 63?80. (revised 3 Nov 2007)

2006-2010 $D = 6$ and $\Delta = 12, 13, 14, D = 3$

2006 Pineda-Villavicencio, Gómez, Miller, Pérez-Rosés. 2009 Gómez

(Δ)	$H_q(K_h)$	*Previous Order*	*New Order*
5	$H_4(K_3)$	2766	**2772**
6	$H_5(K_4)$	7908	**7917**
8	$H_7(K_5)$	39672	**39806**
9	$H_8(K_6)$	75828	**76228**
10	$H_9(K_6)$	134690	**134830**
12	$H_{11}(K_8)$	359646	**359926**
14	$H_{13}(K_{11})$	816186	**818094**

largest graphs $H_q(K_h)$ for $\Delta \leq 14$. D=6

TABLE 2. New large $(\Delta, 3)$-graphs.

(Δ, D)	(12,3)	(13,3)	(14,3)
Previous graph	Q'_8d [12]	Q'_8d [12]	E [18]
Previous order	780	845	912
New graph	Q'_8d^+	Q'_8d^+	Q'_8d^+
New order	786	851	916

New values from G. Exoo (in current table 2023):

- May 12, 2006 (11,2)=104;
- January 28, 2008 (3,7)=196, (3,9)=600
- May 19, 2010 (4,4)=98, (6,3)=111
- May 21, 2010 (5,4)=212

- G. Pineda-Villavicencio, J. Gómez, M. Miller, and H. Pérez-Rosés, New largest graphs of diameter 6, Electron Notes Discrete Math. 24 (2006) 153-160.
- J. Gómez, Some new large Δ, 3)-graphs, Networks 53 (2009) 1-5.
- Gómez, J. On large $(\Delta, 6)$-graphs, Networks 46 (2005), 82-87.
- Gómez, J., I. Pelayo and C. Balbuena, New large graphs with given degree and diameter six, Networks 34 (1999) 154-161 1-5.

2008 wiki E. Loz, H. Pérez-Rosés, G. Pineda-Villavicencio

A new project: The degree/diameter problem for several classes of graphs

1 message

Eval Loz <eval@math.auckland.ac.nz> Wed. Nov 12. 2008 at 9:49 AM

Dear Degree Diameter community,

I would like to publicly announce the project "The degree/diameter problem for several classes of graphs" which is the joint work of Hebert Pérez-Rosés, Guillermo Pineda-Villavicencio and myself. Our goal is to create a clear distinction, and a stable source of information, for different classes of graphs in the degree diameter problem. We also aim to improve results and add new theory.

The first stage in this new exciting project was creating a wiki website containing all the information we have available. This wiki can now be viewed at http://moorebound.indstate.edu/index.php/The_Degree/Diameter_Problem

Creating a wiki was initially suggested in a meeting I had with Geoffrey Exoo last year, for both the DD and Cage problems, and thus the wiki is now located on the Indiana State University server. Future updates and contributions can be added independently by researchers from all over the world, and will be regularly moderated by Hebert, Guillermo and myself. We will also update the wiki as our project progresses. Geoffrey will be updating the Cage pages in the future.

The wiki will be a resource that is continuously maintained, moderated and updated by people who are still active in the area in the future (we were told by Geoffrey that the site will be available also in years to come!).

In the preparation of this data we have used a range of recent publications, new unpublished work and also the online tables maintained by Charles Delorme and Francesc Comellas.

We also have included many new graphs that I found recently, especially in the Cayley and bipartite cases. All the adjacency lists for the bipartite graphs I found of orders less than 20,000 are available at: http://www.eyal.tk/degreediameter/. Complete information on the graphs in terms of quotients and groups will be available in our first publication as a part of this new project.

- http://moorebound.indstate.edu/index.php/The_Degree/Diameter_Problem

2011 Combinatorics Wiki E. Loz, H. Pérez-Rosés, G. Pineda-Villavicencio

http://combinatoricswiki.org/wiki/The_Degree_Diameter_Problem_for_General_Graphs Go JUN JUL OCT

16 captures 25

25 Jul 2011 – 7 Mar 2023 2010 2011 2014

Combinatorics Wiki

d\k	2	3	4	5	6	7	8	9	10
3	10	20	38	70	132	196	336	600	1 250
4	15	41	98	364	740	1 320	3 243	7 575	17 703
5	24	72	212	624	2 772	5 516	17 030	57 840	187 056
6	32	111	390	1 404	7 917	19 383	76 461	307 845	1 253 615
7	50	168	672	2 756	11 988	52 768	249 660	1 223 050	6 007 230
8	57	253	1 100	5 060	39 672	131 137	734 820	4 243 100	24 897 161
9	74	585	1 550	8 200	75 893	279 616	1 686 600	12 123 288	65 866 350
10	91	650	2 286	13 140	134 690	583 083	4 293 452	27 997 191	201 038 922
11	104	715	3 200	19 500	156 864	1 001 268	7 442 328	72 933 102	600 380 000
12	133	786	4 680	29 470	359 772	1 999 500	15 924 326	158 158 875	1 506 252 500
13	162	851	6 560	40 260	531 440	3 322 080	29 927 790	249 155 760	3 077 200 700
14	183	916	8 200	57 837	816 294	6 200 460	55 913 932	600 123 780	7 041 746 081
15	186	1 215	11 712	76 518	1 417 248	8 599 986	90 001 236	1 171 998 164	10 012 349 898
16	198	1 600	14 640	132 496	1 771 560	14 882 658	140 559 416	2 025 125 476	12 951 451 931

- http://web.archive.org/web/20110725185954/http://combinatoricswiki.org/wiki/The_Degree_Diameter_Problem_for_General_Graphs

2009 PhD Guillermo Pineda-Villavicencio

Δ \ D	2	3	4	5	6	7	8	9	10
3	Pe 10	$C_5 * F_4$ 20	vC 38	vC 70	$Exoo$ 132	$Exoo$ 192	$Exoo$ 330	CR^{**} 590	$Cond$ 1250
4	$K_3 * C_5$ 15	$Allwr$ 41	$Exoo$ 96	H_3^ω 364	CG 740	LS 1320	LS 3243	LS 7575	LS 17703
5	$K_3 * X_8$ 24	$Exoo$ 72	Sa 210	LS 624	$PGMP$ 2772	LS 5516	LS 17030	LS 53352	LS 164720
6	$K_4 * X_8$ 32	$Exoo$ 110	LS 390	LS 1404	$PGMP$ 7917	LS 19282	LS 75157	LS 295025	LS 1212117
7	HS 50	$Exoo$ 168	Sa 672	DH 2756	LS 11988	LS 52768	LS 233700	LS 1124990	LS 5311572
8	I_7^ω 57	CM Sa 253	LS 1100	LS 5060	$Gómez$ 39672	LS 130017	LS 714010	LS 4039704	LS 17823532
9	$I_8^\omega d$ 74	Q_8^ω 585	LS 1550	LS 8200	$PGMP$ 75893	LS 270192	LS 1485498	LS 10423212	LS 31466244
10	I_9^ω 91	$Q_8^\omega d$ 650	LS 2223	LS 13140	$Gómez$ 134690	LS 561957	LS 4019736	LS 17304400	LS 104058822
11	$Exoo$ 104	$Q_8^\omega d$ 715	$Q_7(T_4)$ 3200	LS 18700	$H_7(T_4)$ 156864	LS 971028	LS 5941864	LS 62932488	LS 250108668
12	I_{11}^ω 133	$Gómez$ 786	$Q_8^\omega * X_8$ 4680	LS 29470	$PGMP$ 359772	LS 1900464	LS 10423212	LS 104058822	LS 600105100
13	MMS 162	$Gómez$ 851	$Q_9(T_4)$ 6560	LS 39576	$H_9(T_4)$ 531440	LS 2901404	LS 17823532	LS 180002472	LS 1050104118
14	I_{13}^ω 183	$Gómez$ 916	$Q_9(T_5)$ 8200	LS 56790	$PGMP$ 816294	$K_1\Sigma_8^1H_{11}$ 6200460	LS 41894424	LS 450103771	LS 2050103984
15	$I_{13}^\omega d$ 186	$(\otimes Q_{2,4})^\omega$ 1215	$Q_{11}(T_4)$ 11712	LS 74298	$H_{11}(T_4)$ 1417248	LS 8079298	LS 90001236	LS 900207542	$Q_{11}\Theta_4H_{11}$ 4149702144
16	$Exoo$ 198	$(\otimes Q_3)^\omega$ 1600	$Q_{11}(T_5)$ 14640	$(\otimes H_3)^\omega$ 132496	$H_{11}(T_5)$ 1771560	$K_1\Sigma_8^1H_{13}$ 14882658	LS 104518518	LS 1400103920	$Q_{11}d\Sigma_6^1H_{13}$ 7394669856

January 2009

- 2009 G. Pineda-Villavicencio, PhD. Topology of Interconnection Networks with Given Degree and Diameter.

(Δ,D) From 2010. Latest results.

2012-2013. Eduardo Canale $(15, 2)$. Alexis Rodriguez $(6, 9), (9, 5), (9, 8)$.

E. Canale
I just made an addition of 4 vertices, in a non-computer-generated way, to the graph P'_{13}, with diameter 2 and max degree 14 [P'_{13}, quotient of the incidence graph of a of projective plane by a polarity]
The resulting graph has max. degree 15, min. degree 13 and diameter 2
(15,2)=187

A. Rodriguez (M.Eng. thesis, sup. E. Canale).

Voltage graphs from a semidirect product.

(9,5)= 8268, (6,9) = 331387, (9,8) = 1697688

- E. Canale, e-mail Aug. 22, 2012
- Alexis Rodríguez. Tesis de Maestría. U. de la República, Montevideo, Uruguay. Búsquedas masivas de grafos de gran orden con grado y diámetro acotados. Orientador: Eduardo Canale. June 2013.

2018. Jianxiang Cheng. $\Delta = 3$, $D = 8$, $n = 360$

The graph is derived from the symmetric graph on 144 vertices with diameter 7 and girth 8 by a complete pairing of its edges that has a large symmetric group. Let G be the symmetric graph and $\sim$ the pairing relation on its edges.
The graph is constructed as follows:
The vertex set of the new graph H is $V(G) \cup E(G)$.
If $v \in V(G)$, $u \in V(G)$, then they are not connected in H.
If $v \in V(G)$, $u \in E(G)$, then they are connected in H iff $v \in u$ in G.
If $v \in E(G)$, $u \in E(G)$, then they are connected in H iff $v \sim u$ by the pairing relation.
The graph H is not a Cayley graph. It has 3 vertex orbits.

- e-mail, october 16th, 2018

2021. Vlad Pelekhaty. $\Delta = 13$, $D = 3$, $n = 856$

I started with Jose Gomez's Q8'd+ 851(13,3) graph and added a few (odd number of) nodes before “regularizing‘ it by connecting the dangling degrees. I managed to get 856(13,3) with my slow and clucky MATLAB

- e-mail, setember 2021

2024. Some comments on reproducibility

- (7,5) = 2756, found by Hafner in 1994 as $\mathbb{Z}_{52} \rtimes_2 \mathbb{Z}_{53}$, can also be obtained as $\mathbb{Z}_{52} \rtimes_A \mathbb{Z}_{53}, A = 8, 12, 18$
- (8,3) = 223, found by FC / Mitjana in 1994 as $\mathbb{Z}_{11} \rtimes_9 \mathbb{Z}_{23}$, (and in 1997 by Sampels with $A = 3$) can also be obtained as $\mathbb{Z}_{11} \rtimes_A \mathbb{Z}_{23}, A = 2, 8, 13$
- (7,4) = 672, found by Sampels in 1997 as $\mathbb{Z}_6 \rtimes_3 9\mathbb{Z}_{112}$, can also be obtained as $\mathbb{Z}_6 \rtimes_A \mathbb{Z}_{112}, A = 23$
- (7,6) = 11988. found by Loz in 2006 as a voltage graph $\mathbb{Z}_{36} \rtimes_2 \mathbb{Z}_{333}$, with quocient B(0,3) and voltages [(18,108)|(14,61)(34,195)(23,14)]. Can also be obtained (FC) as a Cayley graph associated to $\mathbb{Z}_{36} \rtimes_A \mathbb{Z}_{333}$, with $A = 5, 56, 59$
- (8,4) = 1100. found by Loz in 2006 as a voltage graph $\mathbb{Z}_{20} \rtimes_2 \mathbb{Z}_{55}$, with quocient B(0,4) and voltages [(4, 27)(12, 11)(9,9)(19,11)]. Can also be obtained (FC) as a Cayley graph associated to $\mathbb{Z}_{20} \rtimes_A \mathbb{Z}_{55}$, with $A = 3, 8, 38, 47, 48$
- (9,4) = 1550. found by Loz in 2006 as a voltage graph $\mathbb{Z}_{10} \rtimes_4 \mathbb{Z}_{155}$, with quocient B(1,4) and voltages [(5,0)(1,7)(4,52)(6,136)(1,72)]. Can also be obtained (FC) as a Cayley graph associated to $\mathbb{Z}_{10} \rtimes_A \mathbb{Z}_{155}$, with $A = 39$
- (9,5) = 8268. found by Alexis Rodriguez in 2012 as a voltage graph $\mathbb{Z}_{52} \rtimes_2 \mathbb{Z}_{159}$, with quocient B(1,4) and voltages [(14,41)(47,112)(37,82)(10,113)(26,147)]. Can also be obtained (FC) as a Cayley graph associated to $\mathbb{Z}_{52} \rtimes_A \mathbb{Z}_{159}$, with $A = 41, 50, 71$

2024. New values (FC, 2024)

$(7, 6) = 12264.$ Cayley graph with 12264 vertices and 65318 edges. Semidirect product $\mathbb{Z}_{24} \rtimes_A \mathbb{Z}_{511}$, with $A = 90, 125, 139, 153, 167, 202, 335, 468, \ldots$
(former result with 11988 vertices found by Loz in 2006)

$(8, 5) = 5115.$ Many instances. Cayley graphs with 5115 vertices and 20460 edges. $\mathbb{Z}_{15} \rtimes_A \mathbb{Z}_{196}$, with $A = 20, 49, 69, 71, 113, 152, 169, 196, 235, 236, 267, 289, 317, 324, \ldots$
(former result with 5060 vertices found by Loz in 2006)

$(10, 4) = 2331.$ Several instances. Cayley graphs with 2331 vertices and 11655 edges. Semidirect prod. $\mathbb{Z}_9 \rtimes_A \mathbb{Z}_{259}$, with $A = 44, 81, 123, 256, \ldots$
(former result with 2286 vertices found by Loz in 2006)

$(10, 5) = 13203.$ Cayley graph with 13203 vertices and 66015 edges. Semidirect product $\mathbb{Z}_{81} \rtimes_A \mathbb{Z}_{163}$, with $A = 22, 24, 34, 36, 51, \ldots$
(former result with 13140 vertices found by Loz in 2006)

Possible improvements on this table

Find other new families of Cayley graphs. The work from Marcel Abas (2017), suggests a computer search by considering semidirect products $\mathbb{Z}_M^2 \rtimes_A \mathbb{Z}_N$

Make "clever" computer searches: Select only values of M, N, A that provide the largest number of solutions for $A^N \equiv 1$ and a large group center.

Add a few vertices to known graphs and connect them by computer search -like what was done for entry (13,2) -

Study related problems, e.g. the generalized Moore graph problem: Find the smallest transmission $\sigma = \sum_{u,w \in V} d(u,w)$ of a graph with order n and maximum degree Δ. A "competition" on this order/degree problem and can be found at
`https://research.nii.ac.jp/graphgolf/problem.html`

Contributions from Cerf, Stanton, McKay, M. Sampels, et al.:

- V.G. Cerf, D.D. Cowan, R.C. Mullin, R.G. Stanton. A lower bound on the average shortest path length in regular graphs. Networks 4 (1974), pp 335–342.
- M. Sampels. Vertex-symmetric generalized Moore graphs. Discrete Appl. Math. 138 (2004), pp 195–202.
- B.D. McKay, R.G. Stanton, R. G. The Current Status of the Generalised Moore Graph Problem." In Combinatorial Mathematics VI: Proceedings of the Sixth Australia Conference on Combinatorial Mathematics, Armidale, Australia, August 1978. New York: Springer-Verlag, pp. 21-31, 1979.
- Sloane, N. J. A. Sequences A005007/M0199 and A088933 in "The On-Line Encyclopedia of Integer Sequences."

2023. (Δ,D) table.

With links to details, figures and adjacency lists for many (small order) graphs.

LARGEST KNOWN (Δ,D)-GRAPHS

Last updates: (7,6), (8,5), (10,4),(10,5) Jan-Mar, 2024

Δ \ D	2	3	4	5	6	7	8	9	10
3	10	20	38	70	132	196	360	600	1 250
4	15	41	98	364	740	1 320	3 243	7 575	17 703
5	24	72	212	624	2 772	5 516	17 030	57 840	187 056
6	32	111	390	1 404	7 917	19 383	76 461	331 387	1 253 615
7	50	168	672	2 756	12 264	52 768	249 660	1 223 050	6 007 230
8	57	253	1 100	5 115	39 806	131 137	734 820	4 243 100	24 897 161
9	74	585	1 550	8 268	76 228	279 616	1 697 688	12 123 288	65 866 350
10	91	650	2 331	13 203	134 830	583 083	4 293 452	27 997 191	201 038 922
11	104	715	3 200	19 500	156 864	1 001 268	7 442 328	72 933 102	600 380 000
12	133	786	4 680	29 470	359 926	1 999 500	15 924 326	158 158 875	1 506 252 500
13	162	856	6 560	40 260	531 440	3 322 080	29 927 790	249 155 760	3 077 200 700
14	183	916	8 200	57 837	818 094	6 200 460	55 913 932	600 123 780	7 041 746 081
15	187	1 215	11 712	76 518	1 417 248	8 599 986	90 001 236	1 171 998 164	10 012 349 898
16	200	1 600	14 640	132 496	1 771 560	14 882 658	140 559 416	2 025 125 476	12 951 451 931

- `http://comellas.eu` *an alias for*
- `https://web.mat.upc.edu/francesc.comellas/old-files/delta-d/taula_delta_d.html`
- `http://combinatoricswiki.org/wiki/The_Degree_Diameter_Problem_for_General_Graphs`
- `https://shorturl.at/itVY8` `https://rb.gy/e72c1m` *tables from 1996 to 2017)*