

Dynamical complexity and symplectic integrability.

Abstract. Completely integrable systems should be dynamically simple. Nevertheless there exist C^∞ -integrable geodesic flows with positive topological entropy. On the other hand integrable systems in action-angle form (that is systems on the annulus $\mathbb{A}^\ell = T^*\mathbb{T}^\ell$ defined by Hamiltonian functions which depend only on the action variable) obviously have zero topological entropy. By the σ -union property, one easily deduces that the topological entropy of a completely integrable system is localized on the singular set of its first integral map. We will first give a new definition of integrability (which we call strong integrability), which gives enough control on the singular to guarantee that the topological entropy of a strongly integrable system vanishes.

In the class on strongly integrable systems, it is then possible to define a slower entropy (the freedom index), which amounts to considering the polynomial growth rate of the characteristic covering numbers instead of the exponential ones. The freedom index is infinite when the topological entropy is positive. We will give some natural properties of that index and investigate its behaviour on some very simple examples (action-angle systems and Hamiltonian systems on the two dimensional annulus).

Freedom indices for strongly integrable Hamiltonian systems.

Abstract. Most of known examples of completely integrable systems have zero topological entropy. To investigate their complexity it is therefore necessary to introduce more refined invariants. One first natural attempt is to consider the polynomial growth rate of the covering numbers associated with the iterates of the time-one maps, which we call the freedom index of the system. This index has two major drawbacks : it does not satisfy the variational principle (which states that the topological entropy is the supremum of the metric entropies) and does not admit a σ -union property (which states that the topological entropy on a countable union of invariant domains is the supremum of the entropies on the domains). As a consequence its behaviour is quite difficult to deal with, even for very simple systems.

We introduce another invariant, the weak freedom index, the definition of which is based on Pesin's dynamical dimension theory and thermodynamic formalism, which is closely related to the freedom index and which moreover possesses the σ -union property. The striking fact is that the two indices generally take different values, even for very simple gradient systems on a line, but do coincide for Hamiltonian systems in action-angle form. We take advantage of this property to show how to compute both indices of some classes of integrable systems. We finally show how to extend the scope of the usual dynamical complexity theory to form a hierarchy of new invariants which are relevant in the description of integrable Hamiltonian systems.