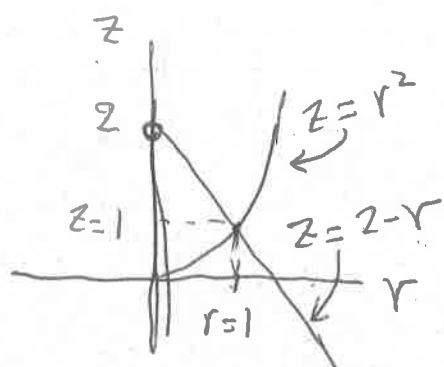


10 Jun 2015, problema 10:



Canvi  $\left. \begin{array}{l} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{array} \right\} \det J_T(r, \theta, z) = rab.$

En aquestes coordenades cilíndriques (adaptades):  
el domini s'escriu:

$$r^2 \leq z \leq 2-r, \quad 0 \leq r \leq 1, \quad 0 \leq \theta \leq 2\pi$$

Lavors

$$V = ab \int_0^{2\pi} d\theta \int_0^1 r dr \int_{r^2}^{2-r} dz = ab \int_0^{2\pi} d\theta \int_0^1 (2r - r^2 - r^3) dr$$

$$= 2\pi \cdot \left(1 - \frac{1}{3} - \frac{1}{4}\right) ab = 2\pi ab \cdot \frac{12-4-3}{12} = \boxed{\frac{5\pi ab}{6}}$$