

IOC 17013 Exercises — Lecture 1

1. The following functions are assumed to be zero for $t < 0$. Show that

(a) the signal $u(t) = 1/(1+t)$ is a power signal and has both 2- and ∞ -norms, but it does not have a 1-norm,

(b) the signal

$$u(t) = \begin{cases} \frac{1}{\sqrt[3]{t}} & \text{if } t \leq 1, \\ 0 & \text{if } t > 1, \end{cases}$$

is a power signal with both 1 and 2-norms, but without ∞ -norm.

2. Consider

$$H(s) = \frac{1}{s^2 + 10^{-6}s + 1}.$$

Compute its ∞ -norm using the grid method and compare with the exact result obtained with the analytic method.

3. Let a system be given by the impulse response $h(t) = e^{-t}\theta(t-2)$. What can you say about the ∞ -norm of the output for an input: a) with 2-norm?, b) with ∞ -norm, c) which is a power signal?
4. Compute the ∞ -norm of

$$H(s) = \frac{s+1}{s^2+s+1}$$

using the state-space representation. Remember that, from a strictly proper transfer function

$$H(s) = \frac{b_ms^m + b_{m-1}s^{m-1} + \dots + b_1s + b_0}{a_ns^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0}, \quad m < n,$$

one can construct (among others) a state space representation given by

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & -\frac{a_2}{a_n} & \dots & -\frac{a_{n-1}}{a_n} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} \frac{b_0}{a_n} \\ \frac{b_1}{a_n} \\ \vdots \\ \frac{b_m}{a_n} \\ 0 \\ \vdots \\ 0 \end{pmatrix}^T.$$

Notice also that, given a representation of $H(s)$, you can get a representation for $\gamma^{-1}H(s)$ simply by replacing C by $\gamma^{-1}C$. Compare with the analytical result obtained directly from the transfer function.