Passive control theory II

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Contents of this lecture

Interconnection and Damping Assignment Passivity Based Control (IDA-PBC)

Magnetic levitation system

Boost converter

DC motor

How to solve quasilinear PDEs

IDA-PBC

Control-as-interconnection has some problems:

Nonlinear PDE for the Casimir functions

Dissipation obstacle

Both problems can be somehow overcomed by considering state-modulated interconnection feedback and controllers with energy function not bounded from below

However, some intuition is lost in the process, so it may be better to go for a more radical approach, which allows much more flexibility, at the expense of immediate physical intuition

Idea: try to find a feedback control such that the closed-loop system is

$$\dot{x} = (J_d(x) - R_d(x))\frac{\partial H_d}{\partial x}(x)$$

$$J_d^T = -J_d$$

$$R_d^T = R_d \geq 0$$

Interconnection assignment   Damping assignment

instead of just

$$\dot{x} = (J(x) - R(x))\frac{\partial H_d}{\partial x}(x)$$

with H_d with a global minimum at the desired regulation point x^*

To do that, one just matches the original dynamics to the desired one

$$(J(x) - R(x))\partial_x H(x) + g(x)\beta(x) = (J_d(x) - R_d(x))\partial_x H_d(x))$$

closed-loop control u

MATCHING EQUATION

The formal result is as follows

Find a (vector) function $K(x)$, a function $\beta(x)$,
a skew-symmetric matrix $J_a(x)$,
and a symmetric, semipositive definite matrix $R_a(x)$ such that

$$(J(x) + J_a(x) - R(x) - R_a(x))K(x) = -(J_a(x) - R_a(x))\frac{\partial H}{\partial x}(x) + g(x)\beta(x)$$

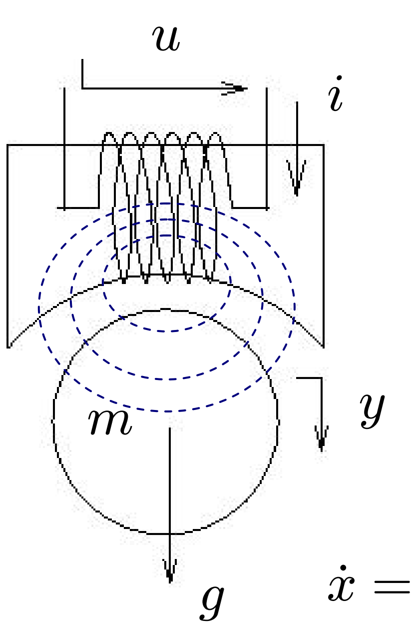
with K the gradient of an scalar, $K(x) = \partial_x H_a(x)$.

Then the closed-loop dynamics with $u = \beta(x)$ is a PHDS with
$H_d = H + H_a$, $J_d = J + J_a$ and $R_d = R + R_a$

with everything else fixed, this is a PDE for $H_a(x)$

However, we can try to select J_a and R_a to make its solution easier

Magnetic levitation system



$$\begin{aligned} \dot{\phi} &= -Ri + u \\ \dot{y} &= v \\ m\dot{v} &= -F_m + mg \end{aligned}$$

$$F_m = -\partial_y W_c(i, y)$$

$$W_c = \frac{1}{2} L(y) i^2$$

$$L(y) = \frac{k}{a + y} \qquad x_1 = \phi,\ x_2 = y,\ x_3 = mv = p$$

$$\dot{x} = \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} - \begin{pmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \frac{\partial H}{\partial x} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$$

$$H(x) = \underbrace{\frac{1}{2k}(a + x_2) x_1^2}_{\text{magnetic co-energy expresed in energy variables}} + \frac{1}{2m} x_3^2 - mgx_2$$

magnetic co-energy expresed in energy variables
(coincides with energy due to the linearity $\phi = L(y)i$)

$$(\nabla H)^T = \begin{pmatrix} x_1 \frac{a+x_2}{k} \\ \frac{1}{2k} x_1^2 - mg \\ \frac{x_3}{m} \end{pmatrix}$$

Given a desired equilibrium point y^*

$$x^* = \begin{pmatrix} \sqrt{2kmg} \\ y^* \\ 0 \end{pmatrix} \qquad u^* = \frac{R}{k} x_1^* (a + x_2^*)$$

Set first $J_a = 0, \; R_a = 0$

$$(J(x) + J_a(x) - R(x) - R_a(x))K(x) = -(J_a(x) - R_a(x)) \frac{\partial H}{\partial x}(x) + g(x)\beta(x)$$

$$\Downarrow$$

$$(J - R)K(x) = g\beta(x)$$

$$\left.\begin{array}{rcl} -RK_1(x) & = & \beta(x) \\ K_3(x) & = & 0 \\ -K_2(x) & = & 0 \end{array}\right\} \Rightarrow \quad H_a(x) = H_a(x_1)$$

Unfortunately

$$\frac{\partial^2 H_d}{\partial x^2}(x) = \begin{pmatrix} \frac{1}{k}(a+x_2) + H_a''(x_1) & \frac{x_1}{k} & 0 \\ \frac{x_1}{k} & 0 & 0 \\ 0 & 0 & \frac{1}{m} \end{pmatrix}$$

has at least one negative eigenvalue no matter which H_a we choose

$\Downarrow$

no minimum at x^*

Let us try something different and put $R_a = 0$ but

$$\mathrm{J}_a = \begin{pmatrix} 0 & 0 & -\alpha \\ 0 & 0 & 0 \\ \alpha & 0 & 0 \end{pmatrix}$$

$$(J(x) + J_a(x) - R(x) - R_a(x))K(x) = -(J_a(x) - R_a(x))\frac{\partial H}{\partial x}(x) + g(x)\beta(x)$$

$$\left.\begin{aligned} -\alpha K_3 - RK_1(x) &= \frac{\alpha}{m}x_3 + \beta(x) \\ K_3(x) &= 0 \\ \alpha K_1(x) - K_2(x) &= -\frac{\alpha}{k}(a + x_2)x_1 \end{aligned}\right\} \Rightarrow H_a = H_a(x_1, x_3)$$

$$u = \beta(x) = RK_1 - \alpha\frac{x_3}{m}$$

$$\alpha\frac{\partial H_a}{\partial x_1} - \frac{\partial H_a}{\partial x_2} = -\alpha\frac{x_1(a + x_2)}{k}$$

This is a quasilinear PDE for H_a and we have to solve it

Method of characteristics

Equations of the form

$$a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u)$$

where $u_x = \partial_x u(x,y)$, $u_y = \partial_y u(x,y)$ are called quasilinear because the derivatives of u appear linearly.

The method of characteristics works as follows.

Construct the following system of ODE for $x(\tau)$, $y(\tau)$, $u(\tau)$

$$\left.\begin{aligned} x'(\tau) &= a(x(\tau),y(\tau),u(\tau)) \\ y'(\tau) &= b(x(\tau),y(\tau),u(\tau)) \\ u'(\tau) &= c(x(\tau),y(\tau),u(\tau)) \end{aligned}\right\}$$

the solutions are called *characteristic curves* and their projections on $u = 0$ are simply called *characteristics*

$$\begin{aligned} x'(\tau) &= a(x(\tau), y(\tau), u(\tau)) \\ y'(\tau) &= b(x(\tau), y(\tau), u(\tau)) \\ u'(\tau) &= c(x(\tau), y(\tau), u(\tau)) \end{aligned}$$

We introduce next a curve of initial conditions

$$(x(0,s), y(0,s), u(0,s))$$

parameterized by s

If the curve of initial conditions does not lie on a characteristic curve, their evolution will generate a two dimensional manifold in $\mathbb{R}^3$

$$(x(\tau,s), y(\tau,s), u(\tau,s))$$

Finally, from

$$\begin{aligned} x &= x(\tau,s) \\ y &= y(\tau,s) \\ u &= u(\tau,s) \end{aligned}$$

we can eliminate τ and s in terms of x and y and obtain the solution $u(x,y)$ to the PDE

The solution depends on arbitrary functions specifying the curve of initial conditions

As an example, consider

$$3u_x + 5u_y = u$$

with an initial curve $(s, 0, f(s))$ where f is arbitrary.

$$\left.\begin{aligned} x' &= 3 \\ y' &= 5 \\ u' &= u \end{aligned}\right\} \longrightarrow \quad \begin{aligned} x &= 3\tau + s \\ y &= 5\tau \\ u &= f(s)e^{\tau} \end{aligned}$$

We get $\tau = y/5$ and $s = x - 3y/5$ and then

$$u(x, y) = f(x - \frac{3y}{5})e^{\frac{y}{5}}$$

Exercise. Solve the PDE for $H_a(x_1, x_2)$ for the levitating system.
It is better to give the initial condition curve in the form $(s, 0, f(s))$.

Boost converter

Consider the averaged model of the boost converter, where we set $u = 1 - S$:

$$\mathcal{J}(u) = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix}, \quad \mathcal{R} = \begin{pmatrix} 1/R & 0 \\ 0 & 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H(x_1, x_2) = \frac{1}{2C} x_1^2 + \frac{1}{2L} x_2^2$$

The control goal is to regulate the load voltage (resistor) at a desired value V_d

input voltage

$$x^* = (CV_d, \frac{LV_d^2}{RE}) \qquad u^* = \frac{E}{V_d} \leq 1$$

output (load) voltage

One can get a controller by setting $J_a = 0$, $R_a = 0$, but a better one can be obtained if $J_a = 0$ but

$$R_a = \begin{pmatrix} -1/R & 0 \\ 0 & r_a \end{pmatrix} \quad \text{so that} \quad R_d = \begin{pmatrix} 0 & 0 \\ 0 & r_a \end{pmatrix}$$

with $r_a > 0$. The IDA-PBC equation is now

$$\begin{pmatrix} 0 & u \\ -u & -r_a \end{pmatrix} \begin{pmatrix} \partial_1 H_a \\ \partial_2 H_a \end{pmatrix} = \begin{pmatrix} -\frac{1}{R} & 0 \\ 0 & r_a \end{pmatrix} \begin{pmatrix} \frac{x_1}{C} \\ \frac{x_2}{L} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} E$$

The standard trick when the control appears in both equations is to isolate the two partial derivatives

$$\begin{aligned} \partial_1 H_a &= \frac{r_a}{RC}\frac{x_1}{u^2} - \frac{r_a}{L}\frac{x_2}{u} - \frac{E}{u} \\ \partial_2 H_a &= -\frac{1}{RC}\frac{x_1}{u} \end{aligned}$$

$$\partial_1 H_a = \frac{r_a}{RC}\frac{x_1}{u^2} - \frac{r_a}{L}\frac{x_2}{u} - \frac{E}{u}$$
$$\partial_2 H_a = -\frac{1}{RC}\frac{x_1}{u}$$

since $\partial_2\partial_1 H_a = \partial_1\partial_2 H_a$,
we get, with $\alpha = 1 - r_a RC/L$,

$$\left(-2r_a x_1 + \frac{r_a RC}{L} u x_2 + ERCu\right)\partial_2 u - x_1 u \partial_1 u + \alpha u^2 = 0$$

This is a PDE of the kind we know how to solve.
However, if we look for solutions of the form $u = u(x_1)$

$$x_1 \partial_1 u = \alpha u$$

with solution, satisfying the appropiate fixed point limit,

$$u(x_1, x_2) = \frac{E}{V_d}\left(\frac{x_1}{x_1^*}\right)^{\alpha}$$ which makes sense provided that $\alpha \geq 0$.

It remains to be checked that one can obtain
an H_d with a minimum at the desired point.

DC motor

Consider a DC motor for which we do not consider
the field coil dynamics (or it has just a permanent magnet).
The system is then 2-dimensional, with port Hamiltonian structure

$$\dot{x} = (J - R)\partial_x H + g + g_u u \qquad H(\lambda, p) = \frac{1}{2L}\lambda^2 + \frac{1}{2J}p^2$$

$$J = \begin{pmatrix} 0 & -K \\ K & 0 \end{pmatrix} \quad R = \begin{pmatrix} r & 0 \\ 0 & b \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ -\tau_L \end{pmatrix} \quad g_u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Assume the control objective is regulation of the mechanical speed to $\omega = \omega_d$.

$$\begin{aligned} i^* &= \frac{1}{K}(b\omega_d + \tau_L) \\ u^* &= ri^* + K\omega_d \end{aligned}$$

Here we adopt a new completely different approach to solve the IDA-PBC equation.

We *impose* an desired Hamiltonian with the correct minimum and then determine J_a and R_a so that the equation is satisfied.

$$H_d = \frac{1}{2L}(\lambda - \lambda^*)^2 + \frac{1}{2J}(p - p^*)^2$$

In fact, in this case it is better to work with the Matching Equation instead of the IDA-PBC one (they are the same)

We go for a completely arbitrary $J_d - R_d = \begin{pmatrix} -r_d & -j_d \\ j_d & -b_d \end{pmatrix}$

The second row of the matching equation imposes

$$j_d(i - i^*) - b_d(\omega - \omega_d) = Ki - b\omega - \tau_L$$

Setting $b_d = b$, and using the equilibrium point expression, $j_d = K$.

Finally, using this and going to the first row of the matching equation,

$$u = -r_d(i - i^*) - ri + K\omega_d$$

where we still have $r_d > 0$ free to tune the controller.

Notice that this controller depends on i^*, which in turn depends on the mechanical torque τ_L.

This makes this controller useless as a regulating speed controller, since it will not be able to do the job if the external torque changes.

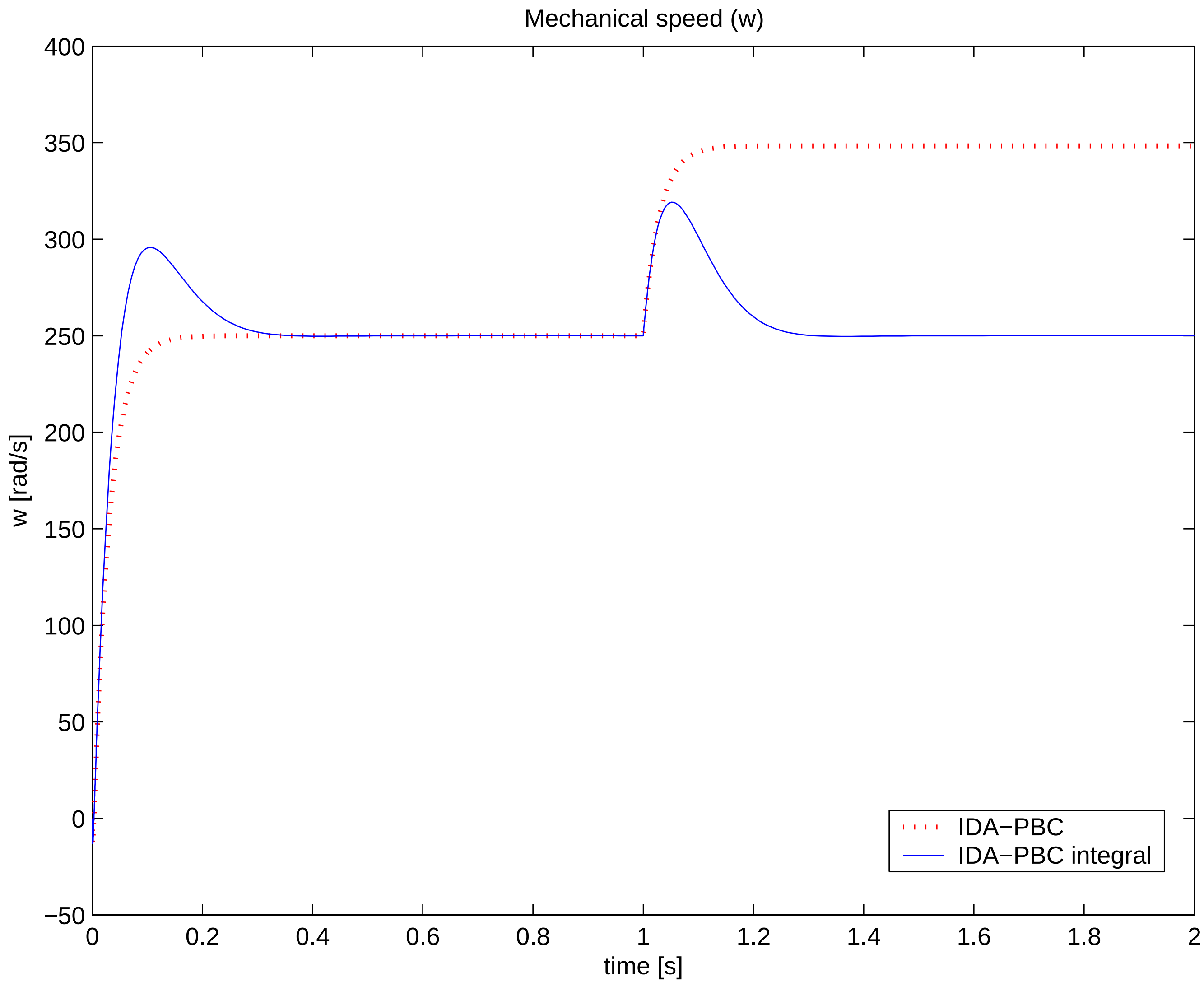
We know from PID theory that this can be solved by adding an integrator:

$$u_i = -r_d(i - i^*) - ri + K\omega_d - \int (\omega - \omega_d)\mathrm{d}t$$

This new controller has been obtained outside the Hamiltonian framework.

→ WORK IN PROGRESS

Controller comparison. $\omega_d = 250$, and τ_L is changed at $t = 1$.



Conclusions

- Energy based control is well suited for energy based modeling.
- Physical structure can be incorporated.
- Open problems and extensions:
  - Robustness.
  - Non regulation problems.
  - Distributed systems: boundary and bulk control.
  - Other ideas.