
MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Linear Numerical Algebra
Activity II.1. Heat transfer between rooms in a building

Note: The deliverable of this assignment must be in *Maple* format, adding brief explanations of every step you perform. The (personal/groupal) way in which you explain it will be positively valued.

We are going to study the heat distribution in a building with some boundary conditions. We assume a rectangular structure of the building, where the North wall is kept at a constant temperature $T_N := 20.0^\circ C$, the East and West wall at $T_E := 15.6^\circ C$ and $T_W := 15.6^\circ C$, respectively, and the South wall at $T_S := 11.4^\circ C$.¹

The physical principle behind our treatment of the heat distribution inside the building lays on the assumption that, as time goes by, the temperature of each room cell varies according its own temperature and that of the neighbouring cells.

To simplify the problem, we first consider the room divided into 6 rectangles as follows:

$$\begin{array}{c}
 T_N \\
 \begin{array}{|c|c|c|}
 \hline
 T_{1,1} & T_{1,2} & T_{1,3} \\
 \hline
 T_{2,1} & T_{2,2} & T_{2,3} \\
 \hline
 \end{array} \\
 T_S
 \end{array}
 \begin{array}{c}
 T_W \quad T_E
 \end{array}$$

assuming uniform temperature inside each rectangle.

We would like to control the temperatures inside the building; that is, $T_{1,1}$, $T_{1,2}$, $T_{1,3}$, $T_{2,1}$, $T_{2,2}$ and $T_{2,3}$. We will solve this problem using basic linear algebra. Our initial conditions are:

$$\begin{aligned}
 \mathcal{T}^{(0)} &= (T_N^{(0)}, T_S^{(0)}, T_E^{(0)}, T_W^{(0)}, T_{1,1}^{(0)}, T_{1,2}^{(0)}, T_{1,3}^{(0)}, T_{2,1}^{(0)}, T_{2,2}^{(0)} \text{ and } T_{2,3}^{(0)})^\top \\
 &= (20.0, 11.4, 15.6, 15.6, 18.3, 18.1, 18.4, 18.3, 18.5, 18.2).
 \end{aligned}$$

¹We are assuming them constant to simplify the problem, but this is a non-natural hypotheses. Later on, we are going to put more realistic constraints.

We suppose that the temperature in each cell after $(n + 1)$ hours is obtained by adding the current temperature of the cell itself divided by 2 with that of each of the neighbouring cells divided by 8. For instance:

$$T_{1,1}^{(n+1)} = \frac{1}{2}T_{1,1}^{(n)} + \frac{1}{8}(T_N + T_{1,2}^{(n)} + T_{2,1}^{(n)} + T_W),$$

where $T_{i,j}^{(n)}$ represents the current temperature of the cell (i, j) and $T_{i,j}^{(n+1)}$ represents this cell temperature one hour later. Observe that, for all $n \geq 1$, $T_N^{(n)}$ satisfies the equation $T_N^{(n)} = T_N^{(n-1)}$ and so, $T_N^{(n)} = T_N$ for all $n \geq 1$. Analogously, $T_S^{(n)} = T_S$, $T_E^{(n)} = T_E$ and $T_W^{(n)} = T_W$ for all $n \geq 1$, since the external temperature is not influenced by that of the room.

We will divide the assignment into three blocks.

Block I. The Maple file `LinearNumericalAlgebra-Activity.mws` contains the corresponding code. You just have to read and understand it, apply it appropriately at each statement and write down the conclusions you get from there.

- (a) Find the equations satisfied by the cells temperatures $T_{1,2}$, $T_{1,3}$, $T_{2,1}$, $T_{2,2}$ and $T_{2,3}$, and rewrite them as a 10×10 linear system with variables: T_N , T_S , T_E , T_W , $T_{1,1}$, $T_{1,2}$, $T_{1,3}$, $T_{2,1}$, $T_{2,2}$ and $T_{2,3}$.
Express it in matrix form:

$$\mathcal{T}^{(n+1)} = A \cdot \mathcal{T}^{(n)},$$

where $\mathcal{T}^{(n)} = (T_N^{(n)}, T_S^{(n)}, T_E^{(n)}, T_W^{(n)}, T_{1,1}^{(n)}, T_{1,2}^{(n)}, T_{1,3}^{(n)}, T_{2,1}^{(n)}, T_{2,2}^{(n)}, T_{2,3}^{(n)})^\top$ and $\mathcal{T}^{(n+1)} = (T_N^{(n+1)}, T_S^{(n+1)}, T_E^{(n+1)}, T_W^{(n+1)}, T_{1,1}^{(n+1)}, T_{1,2}^{(n+1)}, T_{1,3}^{(n+1)}, T_{2,1}^{(n+1)}, T_{2,2}^{(n+1)}, T_{2,3}^{(n+1)})^\top$.

- (b) Check that $\mathcal{T}^2$, the temperatures after two hours, can be obtained from $A^2 \cdot \mathcal{T}^{(0)}$.
Note: In fact, observe that $\mathcal{T}^{(k)} = A^k \cdot \mathcal{T}^{(0)}$. Check it with some examples.
- (c) Using Maple(command `Eigenvectors(A)`), find the eigenvalues and the eigenvectors of A .
- (d) What is the relationship between the eigenvalues and eigenvectors of A and those of A^2 ? Try to obtain a theoretical relation by hand and then check it with the matrix A of this activity.
- (e) Describe the heat distribution of the 6 cells during 24 hours. Plot the temperature profile for each of the 6 interior cells.
- (f) **(optional)** Consider now the same problem with a $2r \times 3r$ cells' grid, assuming tha. As in statement (e), describe the heat distribution of the $6r^2$ cells during 24 hours, for $r = 2$ and $r = 3$. Take one cell and compare the temperature profile for $r = 1, 2, 3$.

Block II. Let us suppose now that we want to install double glasses in the South walls. There are several ways to implement this circumstance. One of them (you can propose other options though) is adding a new cell neighbouring all the previous South cells, thus incorporating a new variable T_{esp} into the system.

$$\begin{array}{c}
 T_N \\
 \begin{array}{|c|c|c|}
 \hline
 T_{1,1} & T_{1,2} & T_{1,3} \\
 \hline
 T_{2,1} & T_{2,2} & T_{2,3} \\
 \hline
 \mathbf{T}_{\text{esp}} & & \\
 \hline
 \end{array} \\
 T_S
 \end{array}
 \begin{array}{l}
 \\
 T_E
 \end{array}$$

- (a) State the new set of equations and compute the heat distribution after 12 and 24 hours taking $T_{\text{esp}}^{(0)} = 18.4^\circ C$.
- (b) Compare them with those obtained in Block I (e).

Block III. **(optional)** Simulate the heat evolution when introducing on the South wall the variable temperature profile used in Activity I.1:

$$T_A(t) = 8 + 8 \sin(1/12 \pi (t - 11)),$$

in Celsius degrees, where t units are hours.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Linear Numerical Algebra
Activity II.2. Design of a lighting configuration.

Note: The deliverable of this assignment must be in *Maple* format, adding brief explanations of every step you perform. The (personal/groupal) way in which you explain it will be positively valued.

We will see an application of linear algebra to the design of a lighting configuration in a building project.

We assume to have four points of light at the ceiling of a hall, located at the vertices of a rectangle whose sides measure A and B units of length, as indicated in the figure. We denote by i_1, i_2, i_3 and i_4 the lighting intensity at each of the four points of light.

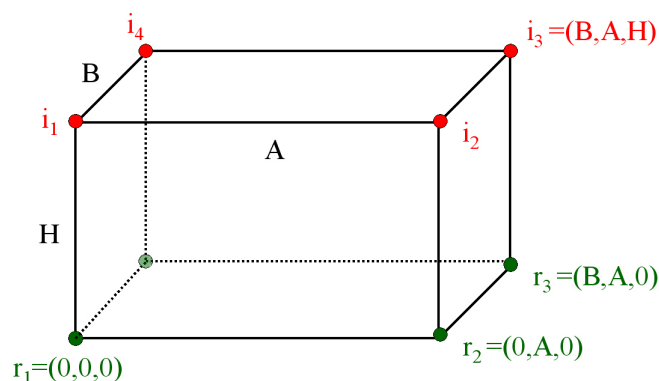


Figura 1: Location of the points of light $i_j, j = 1, \dots, 4$, and the sensors $r_j, j = 1, \dots, 4$.

The purpose is guessing how do you have to regulate these intensities in order to reach appropriate lighting intensities on the floor of the hall. For this purpose, we have installed several sensors (luminometers) on the floor just below each point of light; we denote by r_1, r_2, r_3 and r_4 the light intensities received by each sensor. Obviously, the values r_1, r_2, r_3 and r_4 depend upon the intensities i_1, i_2, i_3 and i_4 emitted by the light focuses. On the other hand, assuming the room's height is H units of length, it is easy to compute the intensity detected by the sensors by means of the *Lambert's cosine law* which claims that the luminous intensity received is proportional to the cosine of the incidence angle.

For instance, the intensity received by the first sensor, r_1 , will be the addition of the intensities received from each point of light. From the first focus, it receives

$i_1 \cos(0^\circ) = i_1$; from the second one, $i_2 \cos(\alpha)$, where the incidence angle α is the quotient between the adjacent leg and the hypotenuse of the triangle formed by the ray from the second point of light to the sensor; it is not difficult to see that it satisfies $\cos(\alpha) = H/\sqrt{H^2 + A^2}$.

Analogously, we can compute the other two intensities and we will get:

$$r_1 = i_1 + \frac{H}{\sqrt{H^2 + A^2}} i_2 + \frac{H}{\sqrt{H^2 + A^2 + B^2}} i_3 + \frac{H}{\sqrt{H^2 + B^2}} i_4.$$

Observe that this relation is linear, that is, r_1 is a linear combination of i_1, i_2, i_3 and i_4 since A, B and H are supposed to be known data.

Solve the following statement using $H = 2.1$, $A = 11.5$ and $B = 7.6$. Use exact values whenever is possible.

Step 1: Four focuses configuration.

- (a) Find the linear equations that determine r_2, r_3 and r_4 . Write the resulting system as $A \vec{i} = \vec{r}$, where $\vec{i} = (i_1, i_2, i_3, i_4)$ and $\vec{r} = (r_1, r_2, r_3, r_4)$.
- (b) Find the LU decomposition of the matrix A .
- (c) Use the LU decomposition to find the necessary intensity from the four points of light in order to obtain a configuration that projects more luminous intensity on the sensor r_1 : $\vec{r} = (0.8, 0.6, 0.6, 0.6)$.
- (d) With the intensities computed in the previous statement, compute the luminous intensity received at a generic point on the floor with coordinates $(x, y, 0)$. Plot these values using the commands `densityplot` and/or `contourplot` of Maple.
- (e) Suppose now that focus number 2 breaks, so that $i_2 = 0$. could we maintain the same intensity?

Step 2: Five focuses configuration.

Assuming that focus number 2 is restored, suppose that we add a fifth lighting point on the ceiling, at the point Q_H located at $(B/2, A/2, H)$; recompute the distribution of intensities to get again $\vec{r} = (0.8, 0.6, 0.6, 0.6)$. Is there a unique solution? Why?

Step 3: Five sensors configuration.

We reconsider the situation with four points of light, as in Step 1, but we add a fifth sensor at the point Q_0 located at $(B/2, A/2, 0)$; denote by r_5 the luminous intensity received by this sensor. Explain why some lighting configurations on the five sensors, $\vec{r} = (r_1, r_2, r_3, r_4, r_5)$, are not possible anymore; provide an example such that $r_j \neq 0$, for $j = 1, 2, 3, 4, 5$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group A
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of copper, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 2, \quad u(0, x) = 2(1 + 8x(x - L)/L^2)$$

where $L = 3$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group B
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of aluminium, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 3, \quad u(0, x) = 3(1 + 8x(x - L)/L^2)$$

where $L = 2$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group C
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of PVC, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 2, \quad u(0, x) = 2(1 + 8x(x - L)/L^2)$$

where $L = 5$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group D
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of pine wood, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 5, \quad u(0, x) = 5(1 + 8x(x - L)/L^2)$$

where $L = 2$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group E
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of cork, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 3, \quad u(0, x) = 3(1 + 8x(x - L)/L^2)$$

where $L = 5$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group F
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of concrete, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 5, \quad u(0, x) = 5(1 + 8x(x - L)/L^2)$$

where $L = 3$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group G
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of glass fiber, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 7, \quad u(0, x) = 7(1 + 8x(x - L)/L^2)$$

where $L = 5$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group H
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of steel, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 2, \quad u(0, x) = 2(1 + 8x(x - L)/L^2)$$

where $L = 7$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group I
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of glass fiber, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 4, \quad u(0, x) = 4(1 + 8x(x - L)/L^2)$$

where $L = 3$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group J
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of steel, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 5, \quad u(0, x) = 5(1 + 8x(x - L)/L^2)$$

where $L = 4$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.

MODELS DE PREDICCIÓ EN L'EDIFICACIÓ
Heat equation. Group K
Activity III. Integrating the 1D heat equation

Note: This assignment must be delivered in *Maple* or **Matlab** format, inserting brief explanations to guide the reader. The way in which you explain it will be also evaluated.

We want to study the heat transmission in a rod made of pine wood, in transient regimes (see class notes of **Physics** to recall the concepts).

We will use a numerical method to integrate the heat equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$

subject to boundary conditions

$$u(0) = u(L) = 3, \quad u(0, x) = 3(1 + 8x(x - L)/L^2)$$

where $L = 4$ m is the length of the rod and α is the thermal diffusivity (in m^2/h , see Table on the **Physics** notes).

The purpose is applying the Implicit Euler method explained in class (see notes on **pdf**).

1. **Compute the solutions for $t \in [0, T]$ h, where $T = 24$ hours, $h_0 = 0.1 L$ and $k = 0.01 T$.**
2. Compute the solutions for $t \in [0, T]$ h, where $T = 24$ **hours**. Use different h steps in a decreasing way from $h_0 = 0.1 L$ (already computed in the previous statement), using $h_j = 0.1 h_{j-1}$ for $j > 0$, down to a value h_r that you have to guess such that

$$\max_{x \in [0, L]} |u^{(r)}(T, x) - u^{(r-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is the numerical solution obtained with $h = h_r$ for $j > 0$. Use the constant time resolution $k = 0.01 T$.

3. Give a comprehensive integrated plot of $u(t, x)$ for $t \in [0, T]$.
4. Provide an interpretation of the results obtained.

5. **(optional)** Repeat the previous statements fixing the mesh resolution at $h = h_0$ and diminishing the time resolution k from $k_0 = 0.1$ **T**, using $k_j = 0.1 k_{j-1}$ for $j > 0$, down to a value k_s such that

$$\max_{x \in [0, L]} |u^{(s)}(T, x) - u^{(s-1)}(T, x)| < \epsilon,$$

where $\epsilon = 10^{-3}$ and $u^{(j)}(t, x)$ is now the numerical solution obtained with $k = k_j$ for $j > 0$.