Enumerative Combinatorics M04

Problem Set 2

1.

(i) How many 11-letter words can be formed with the letters `ABRACADABRA`? How many of those do not have two consecutive `R`'s?

(ii) In how many ways can we distribute 6 distinguishable balls in 10 numbered boxes? And if no box can have more than one ball?

(iii) The same as before, but with undistinguishable balls.

2. How many paths are there in the plane from $(0,0)$ to (a,b) using steps of the form $(1,0)$ and $(0,1)$? Generalize to higher dimensions.

3. Find the number of surjective maps from $[n]$ onto $[k]$. Deduce the number of bijections of $[n]$ onto itself.

4. In how many ways can we choose k non-consecutive integers from $[n]$? And if in addition we consider that 1 and n are consecutive?

5. Define:

$$
\begin{aligned}
&A_{2i-1} = \{\pi \in \mathcal{S}_n : \pi(i) = i\} && 1 \leqslant i \leqslant n \\
&A_{2i} = \{\pi \in \mathcal{S}_n : \pi(i) = i+1\} && 1 \leqslant i \leqslant n-1 \\
&A_{2n} = \{\pi \in \mathcal{S}_n : \pi(n) = 1\}
\end{aligned}
$$

Use these sets, PIE, and the previous problem (or otherwise) to find the cardinality of the set $X = \{\pi \in \mathcal{S}_n : \pi(i) \neq i, i+1 \text{ for all } 1 \leqslant i \leqslant n-1 \text{ and } \pi(n) \neq n, 1\}$.

As a corollary, solve the *problème des ménages*:

> *n couples (n men and n women) are to sit in a round table. In how many ways can this be done if men and women alternate but no couple sits side by side?*

AdM